Adaptive Linear Interference Suppression for Packet DS-CDMA

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Abstract

The performance of adaptive linear interference suppression is studied in the context of packet DS-CDMA. A multi-cell system is assumed with stochastic arrivals and departures of asynchronous users, and additive Gaussian noise as the only channel impairment. Interference suppression is achieved with a tapped-delay line filter, where the filter spans a single symbol interval. Adaptive algorithms considered include the stochastic gradient (LMS), exponentially-weighted Least Squares (LS), block LS, and a reduced-rank LS algorithm. The reduced-rank LS algorithm first projects the received signal onto a signal subspace spanned by eigenvectors of the averaged outer product matrix of received vectors. The purpose of the projection is to eliminate low-level background interference and noise. Both decision-directed and blind algorithms, which do not require a training sequence, are compared. Computer simulation is used to obtain error rates as a function of traffic load, and algorithm and system parameters (including timing offset). Results indicate that the adaptive algorithms offer a significant increase in capacity (nearly a factor of two at moderate error rates), and are insensitive to variations in received power over the user population.

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1 Introduction

Interference suppression using the Minimum Mean Squared Error (MMSE) performance criterion has been proposed for Direct Sequence (DS)-Code-Division Multiple Access (CDMA) in [1] - [8]. In [3] - [7] it is observed that the MMSE detector for a particular user can be implemented as a tapped-delay line, analogous to the (fractionally-spaced) linear equalizer for a dispersive single-user channel.

When implemented as a tapped-delay line, the linear MMSE detector for DS-CDMA has the following attractive properties:

- It is robust with respect to strong interference.
- It can be adapted using standard adaptive filtering algorithms (i.e., stochastic gradient or least squares).
- Adaptation requires either an initial training sequence, or knowledge of the signature waveform of the desired user [9]. Estimates of amplitudes and relative phases are not needed.
- The performance degrades gracefully as the number of (equal power) users increases.

The linear MMSE detector is less complex and easier to adapt than many of the multi-user detectors previously proposed (e.g., see [10]). This detector might therefore help to alleviate the stringent requirements on power control in DS-CDMA. However, time-varying channel impairments, such as changing interference and fading, will compromise the performance of the adaptive algorithm used to estimate the filter coefficients.

In this paper we study, via computer simulation, the performance of adaptive interference suppression algorithms in the context of a cellular system model. Only the reverse link is considered. Other results illustrating the performance of adaptive receivers for DS-CDMA, based on the MMSE criterion, have been presented in [3], [5], [7], and [11]. This work differs from prior work in the following important ways:

1. A packet data scenario is considered in which all active users (including the desired user) transmit bursts of data, each of which contains a random (exponentially distributed) number of symbols.
2. The total traffic load in the cell cluster, measured in Erlangs, is much greater than the processing gain.
3. Both intra- and other-cell users are modeled.
4. The performance of a variety of adaptive algorithms are compared with the conventional matched filter receiver.

In our model, users appear at random locations (chosen from a uniform distribution) within the cell cluster. The only difference between intra- and other-cell users is the way in which the received power is computed. The received power for each intra-cell user is selected from a log-normal distribution, corresponding to imperfect power control [12]. Each other-cell user is assumed to be power-controlled by the adjacent base station [13]. Here we ignore short-term power variations due to fading and power control dynamics. That is, the received power for each packet is assumed to be constant. We also do not model phase variations due to frequency offset or Doppler shift associated with high-tier mobility. Recent results which show the performance of adaptive algorithms in a packet DS-CDMA scenario with flat fading are presented in [14].

For each type of receiver (i.e., adaptive algorithm), the error rate is shown as a function of traffic load (measured in Erlangs/cell [15]). All receivers consist of a tapped-delay line filter that spans a single symbol interval [6]. In addition to the matched-filter, the following adaptive algorithms for estimating the filter coefficients are considered: (1) stochastic gradient (LMS), (2) exponentially-weighted Recursive Least Squares (RLS), (3) Block LS (BLS), and (4) Reduced-Rank LS (RRLS). We consider both decision-directed and blind (orthogonally-anchored [9]) versions of the LS algorithms. Decision-directed algorithms require a training sequence for initial adaptation, whereas the blind versions require an estimate of the received signature waveform corresonding to the desired user. The decision-directed Block LS (BLS) algorithm relies on an iterative scheme to estimate the appropriate decisions [16].

An RRLS algorithm first projects the received signal vectors onto a signal subspace. The projected vectors are then used to compute the interference suppression filter. Projecting the received signal vectors onto the subspace spanned by the strongest signal components is known as "principle components" analysis, and has been studied extensively in other signal processing applications [17], [18], [19]. It has been recently pointed out that this projection is suboptimal in a least squares sense [20]. For both principle components analysis and the optimal (LS) subspace projection, the basis vectors of the subspace are a subset of eigenvectors of the averaged outer product matrix of received vectors. For the DS-CDMA application the objective of this projection is to reduce the influence of low-level background interference and noise. Also, reducing the number of adaptive coefficients enables faster tracking of interference transients. Related work on reduced-rank filtering for CDMA interference suppression is presented in [21], where the signal subspace is estimated recursively.

Our results indicate that at moderate error rates the adaptive receivers provide a significant increase in capacity (nearly a factor of two) relative to the matched filter receiver. Furthermore, performance is insensitive to variations in received power from different users. The BLS algorithm generally performs best, provided that the number of data vectors used to estimate the filter coefficients (block length) is sufficiently large. The RRLS algorithm enables the use of relatively short block lengths with modest performance degradation.

We also examine the effect of timing offset. (Performance results for linear non-adaptive multiuser receivers in the presence of timing offset are reported in [22] and [23].) Results show that the performance of the adaptive algorithms degrades gracefully with timing offset. Also, the gain in capacity offered by the adaptive algorithms relative to the matched filter is not a sensitive function of timing offset.

In the next section we present the DS-CDMA cellular model. Section 3 presents the adaptive interference suppression algorithms, and Section 4 presents our numerical results.

2 System Model

We focus on the reverse link of a particular cell in a multi-cell system. Each active user is assumed to transmit a baseband signal

$$x_k(t) = \sum_i A_i b_{i,k} s_k(t - iT - \tau_k) \tag{1}$$

where $b_{i,k}$ is the ith symbol transmitted by user k, $s_k(t)$ is the spreading waveform associated with user k, and τ_k and A_k are respectively the delay and amplitude associated with user k. Throughout this paper we will assume BPSK modulation with coherent detection corresponding to $b_{i,k} \in \{\pm 1\}$. For DS-CDMA,

$$s_k(t) = \sum_{i=1}^{N-1} a_{i,k} \Psi(t - iT_c) \tag{2}$$

where $a_{i,k} \in \{\pm 1/\sqrt{N}\}$, $i = 0 \ldots N-1$, is the spreading sequence for user k, $\Psi(t)$ is the chip waveform, T_c is the chip duration, and $N = T/T_c$ is the processing gain. The spreading sequence for each user is chosen randomly from a uniform distribution. The adaptive algorithms considered in the next section assume that the same spreading waveform is used for each symbol. A short spreading sequence is a requirement for linear (time-domain) interference suppression. In contrast, a very long spreading sequence is used in the current Interim Standard (IS)-95 DS-CDMA air interface. The results for the matched filter (used in IS-95) presented in the next section assume a short spreading sequence. (This is because simulations with long spreading sequences take much longer to run.) For the uncoded results shown in Section 4, the time-averaged performance of the matched filter is independent of whether long or short spreading sequences are used. This is due to the fact that the user population is different from packet to packet, so that the variations in performance observed in [24] and [25] are averaged out.[1]

Let $\mathbf{r}_i$ be the N-vector containing samples at the output of a chip-matched filter during the ith transmitted symbol, assuming that the receiver is synchronized to the

[1] With coding the use of short vs. long spreading sequences may produce different results, depending on the burstiness of uncoded errors.

desired user. Letting the user to be detected correspond to $k = 1$, and ignoring multipath, we can write

$$\mathbf{r}_i = b_{i,1}\mathbf{s}_1 + \sum_{k=2}^{K} A_k(b_{i,k}\mathbf{s}_k^{(r)} + b_{i-1,k}\mathbf{s}_k^{(l)}) + \mathbf{n}_i \tag{3}$$

where $\mathbf{s}_1$ is the spreading sequence associated with user 1, $\mathbf{s}_k^{(l)}$ and $\mathbf{s}_k^{(r)}$ are the N-vectors obtained at the output of the chip-matched filter in response to user k's shifted waveforms $s_k(t + T - \tau_k)$ and $s_k(t - \tau_k)$, respectively, and $\mathbf{n}_i$ is the vector of noise samples, assumed to be white with covariance $\sigma^2\mathbf{I}$. Because the users are asynchronous, each interferer contributes two vectors to the sum in (3). Expressions for the vectors $\mathbf{s}_k^{(l)}$ and $\mathbf{s}_k^{(r)}$ in terms of the delay τ_k and the chip shape are given in [6]. The numerical results in Section 4 assume rectangular chip shapes.

All users transmit data packets, which contain an exponentially distributed number of symbols. (Packets transmitted by the desired user are padded so that they contain an integer number of blocks for the BLS algorithms.) The mean packet length is L symbols. Packets arrive according to a Poisson process with rate λ. (This traffic model is more appropriate for data transmission than for voice calls.) At any time t the number of active users in the system K is a Poisson random variable with mean λL.

Each time the desired user transmits a new packet, it is likely that the set of active interferers has changed. Furthermore, because of the random delay between packets, each interferer is shifted randomly relative to the desired user. The optimal (MMSE) receiver tapped-delay line coefficients can therefore change significantly from packet to packet. The model simulated assumes that for each desired packet there is a different set of users with random delays (selected from a uniform distribution) and random locations. The number of users at the onset of the packet is a Poisson random variable, and the spreading code for each user is selected from a uniform distribution. (Users subsequently arrive and depart during the packet, as previously explained.)

The only distinction between intra-cell users and other-cell users is the way in which the received power is computed. For intra-cell users, the received power is given by

$$A_k^2 = 10^{\xi/10} \tag{4}$$

where ξ is Gaussian with mean zero and standard deviation σ in dB. This models the situation in which the powers of the intra-cell users are controlled by the same base station. The standard deviation σ represents the strictness of the power control [12].

If the location of the other-cell user is displaced by a vector $\mathbf{x}$ from the adjacent base station, then the received power at the base station of interest is [13]

$$A_k^2 = P_0 \frac{\|\mathbf{x}\|^4}{\|\mathbf{x} + 2\mathbf{r}\|^4} 10^{(\xi_0 - \xi_k)/10} \tag{5}$$

where P_0 is the nominal transmitted power, $2\mathbf{r}$ is the vector from the base station of interest to the adjacent base station, and ξ_0 and ξ_k are independent Gaussian random variables, each assumed to have mean zero and a standard deviation of 8 dB. Also, A_k is constrained to be less than one, since otherwise, the user would be assigned to the base station of interest.

For the numerical results presented in Section 4 the received power for each user is a constant for the duration of the call. In particular, we do not model power variations caused by dynamic power control or fading. In addition, we assume perfect carrier recovery with coherent detection.

3 Adaptive Receivers

All of the receivers considered can be represented by a vector $\mathbf{c}_i$, which is used to compute the estimated bit at time i:

$$\hat{b}_{i,1} = \mathrm{sign}(\mathbf{c}_i'\mathbf{r}_i). \tag{6}$$

The adaptive receivers will be compared with the (non-adaptive) matched filter receiver for which $\mathbf{c}_i = \mathbf{s}_1$ for all i.

The adaptive algorithms we consider are based on the MMSE criterion. Specifically, we wish to choose $\mathbf{c}_i$ to minimize

$$\mathrm{MSE} = E\{(b_{i,1} - \mathbf{c}_i'\mathbf{r}_i)^2\} \tag{7}$$

The MMSE solution for $\mathbf{c}_i$ is

$$\mathbf{c}_i = \mathbf{R}^{-1}\mathbf{p} \tag{8}$$

where

$$\mathbf{R}_i = E\{\mathbf{r}_i\mathbf{r}_i'\} = \mathbf{s}_1\mathbf{s}_1' + \sum_k A_k^2(\mathbf{s}_k^{(r)}\mathbf{s}_k^{(r)'} + \mathbf{s}_k^{(l)}\mathbf{s}_k^{(l)'}) + \sigma^2\mathbf{I} \tag{9}$$

and $\mathbf{p} = E\{b_{i,1}\mathbf{r}_i\} = \mathbf{s}_1$. In the presence of time-varying interference the terms in the sum in (9) change, so that $\mathbf{R}_i$ and $\mathbf{c}_i$ are time-varying.

3.1 Stochastic Gradient

The simplest adaptive algorithm considered is the (normalized) stochastic gradient, or LMS algorithm [26], [27]

$$\mathbf{c}_{i+1} = \mathbf{c}_i + \overline{\mu} e_i \mathbf{r}_i \tag{10}$$

where the error $e_i = \hat{b}_{i,1} - \mathbf{c}_i{}'\mathbf{r}_i$, and the stepsize $\overline{\mu} = \kappa/\hat{E}_i$, where κ is a constant (< 1) and $\hat{E}_i = (1-\mu)\hat{E}_{i-1} + \mu\|\mathbf{r}_i\|^2$ is an estimate of the received signal energy. (μ is a constant close to one.) The algorithm can be initially adapted with a training sequence (i.e., $\{b_{i,1}\}$ is known at the receiver), and can subsequently switch to decision-directed mode (i.e., $\hat{b}_{i,1}$ is given by (6)). Although simple, the algorithm generally converges slowly. Simulation results have shown that, in fact, the LMS algorithm is unable to track the interference environment, assuming a moderate traffic load, and performs much worse than the matched-filter receiver.

3.2 Least Squares (LS)

An alternative to stochastic gradient algorithms is to select $\mathbf{c}$ to minimize the least squares (LS) cost function

$$C_{LS} = \sum_{i=i-B+1}^{i} w^{i-l}(\hat{b}_{l,1} - \mathbf{c}'\mathbf{r}_l)^2 \tag{11}$$

where the limits of the sum represent the window of interest (B is the block length), and w is an exponential weighting factor. The $\mathbf{c}$ which minimizes C_{LS} at time i is

$$\mathbf{c}_i = \hat{\mathbf{R}}_i^{-1}\hat{\mathbf{p}}_i \tag{12}$$

where

$$\hat{\mathbf{R}}_i = \sum_{l=i-B+1}^{i} w^{i-l}\mathbf{r}_l\mathbf{r}_l{}' \tag{13}$$

and

$$\hat{\mathbf{p}}_i = \sum_{l=i-B+1}^{i} w^{i-l}\hat{b}_l\mathbf{r}_l \tag{14}$$

"Recursive" LS (RLS) means that $B = i$, and the solution $\mathbf{c}_i$ is computed for each i. Exponential weighting is needed in a time-varying environment to discount past data. The matrix inverse $\mathbf{R}_i^{-1}$ can be propagated in time via the matrix inversion lemma ([26], Sec. 13.2). To avoid problems with numerical instability, the algorithm can be periodically reinitialized by directly inverting $\mathbf{R}_i$. As with the stochastic gradient algorithm, the RLS algorithm can be run initially with a training sequence, and can subsequently switch to decision-directed mode.

"Block" LS (BLS) means that $\mathbf{c}$ is computed for each successive block of B received vectors. In this case the discount factor $w = 1$. A decision-directed BLS algorithm requires the symbol estimates $\{\hat{b}_{i,1}\}$, $i = i - B + 1, \ldots, i$. This is a problem since a change in the set of interferers which occurs during a block may cause a substantial change in the optimal filter $\mathbf{c}$ from one block to the next. In that case, decisions based on the vector $\mathbf{c}$ from the preceding block are likely to be unreliable. This is in contrast to the RLS algorithm where the LS solution does not change significantly from one symbol to the next.

The estimates $\{\hat{b}_{i,1}\}$ can be obtained via the following iterative approach [16]:

1. Initialize $\mathbf{c}$ as the $\mathbf{c}$ computed for the preceding block. A training sequence is used in the first block.

2. Compute the sequence $\{\hat{b}_{i,1}\}$ from (6).

3. Compute the $\mathbf{c}$ which minimizes C_{LS}.

4. Recompute the sequence $\{\hat{b}_{i,1}\}$.

5. Iterate steps 3 and 4 until the estimated symbol sequence does not change.

At low to modest error rates, this algorithms requires very few iterations (i.e., < 3) to converge. At high error rates ($> 15\%$) the algorithm does not always converge, so that an upper limit on the number of iterations is imposed. Observe that the matrix inverse $\mathbf{R}_i^{-1}$, which appears in (12), only needs to be computed once per block.

3.3 Blind Algorithms

A drawback of decision-directed techniques is that they can become unstable in the presence of large transients. For example, after the appearance of a new strong user, the decisions may be unreliable, which causes the algorithm to lose track of the desired signal. This problem is especially troublesome for the decision-directed BLS algorithm just described. When this problem occurs, either a new training sequence must be transmitted, or the algorithm must switch to a blind mode which does not make explicit use of the estimated symbols.

A blind estimation algorithm for the MMSE filter was proposed in [9]. The vector $\mathbf{c}$ is expressed as the sum

$$\mathbf{c}_i = \mathbf{s}_1 + \mathbf{x}_i \tag{15}$$

where $\mathbf{x}_i$ is constrained to be orthogonal to $\mathbf{s}_1$ (the *anchor*). Selecting $\mathbf{x}_i$ to minimize the variance of the output $E[(\mathbf{c}_i'\mathbf{r}_i)^2]$ also minimizes MSE. An LS algorithm based on this approach selects $\mathbf{x}_i$ to minimize the cost function

$$C_{BL} = \sum_{l=i-B+1}^{i} w^{i-l}(\mathbf{c}_l'\mathbf{r}_l)^2 \tag{16}$$

where from (15) and the orthogonality constraint, $\mathbf{c}_i'\mathbf{s}_1 = 1$. It is easily shown that the solution is

$$\mathbf{c}_i = \kappa \hat{\mathbf{R}}_i^{-1}\mathbf{s}_1 \tag{17}$$

where $\hat{\mathbf{R}}_i$ is given by (13) and $\kappa = 1/(\mathbf{s}_1'\hat{\mathbf{R}}_i^{-1}\mathbf{s}_1)$. Note that this algorithm can be implemented recursively with exponential weighting, or block-by-block.

It was observed in [9] that if the received signal vector corresponding to the desired user is different from $\mathbf{s}_1$ (say, due to multipath and/or timing offset), then minimizing the cost function C_{BL} suppresses the desired signal as well as the interference. In that case, it is necessary to constrain the norm of the coefficient vector $\mathbf{c}_i$. This is accomplished by adding a diagonal matrix to the estimate $\hat{\mathbf{R}}_i$:

$$\hat{\mathbf{R}}_i = \sum_{l=i-B+1}^{i} w^{i-l}\mathbf{r}_l\mathbf{r}_l' + \delta\mathbf{I} \tag{18}$$

where δ is a small constant which determines an upper bound on the norm of $\mathbf{c}_i$. This modification for computing $\mathbf{c}_i$ in (17) was used to generate the results for timing offset in Section 4.

3.4 Reduced-Rank LS

A Reduced-Rank LS (RRLS) algorithm first projects each received vector $\mathbf{r}_i$ onto a lower-dimensional subspace before further processing. The primary motivation for this projection is that tracking and convergence performance of an adaptive algorithm degrade as the number of coefficients increases [27], [26]. By reducing the number of adaptive coefficients (i.e., the dimension of $\mathbf{c}$), an adaptive algorithm is better able to track interference transients. This is especially true when the channels or interference vary rapidly, and short data blocks are needed to estimate the filter coefficients. Of course, the disadvantage in projecting onto a lower-dimensional subspace is that the MMSE generally increases as the dimension D decreases.

Reduced-rank techniques have been extensively studied in other signal processing applications [17] - [20]. The typical objective is to project the received data onto the *signal* subspace. In this way, no useful information is lost by the projection. (In other words, the MMSE associated with the reduced-rank solution is no greater than that for the full-rank solution).

For the asynchronous DS-CDMA application considered here, the dimension of the signal subspace is generally twice the number of users. However, when other-cell interference is taken into account, this no longer applies since twice the number of active users typically exceeds the processing gain. Consequently, the MMSE is expected to increase as the dimension D decreases from N. However, the dimension of the subspace spanned by *strong* (i.e., intra-cell) interferers is expected to be less than N for moderate traffic loads. By projecting onto this subspace the improvement in tracking due to the decrease in the number of adaptive coefficients may offset the increase in associated MMSE. The results in Section 4 indicate that RRLS gives a significant improvement in performance relative to full-rank BLS when the block size B is relatively small.

Let $\mathbf{S}_D$ be the $N \times D$ matrix with column vectors which are an orthonormal basis for a D-dimensional subspace, where $D < N$. The projected received vector corresponding to symbol i is then given by

$$\tilde{\mathbf{r}}_i = \mathbf{S}'_D \mathbf{r}_i \tag{19}$$

(In what follows, all projected variables are denoted with a "tilde".)

The sequence of projected received vectors $\{\tilde{\mathbf{r}}_i\}$ is the input to a tapped-delay line filter, represented by the D-vector $\tilde{\mathbf{c}}_i$ for symbol i. The filter output corresponding to the ith transmitted symbol is

$$y_i = \tilde{\mathbf{c}}'_i \tilde{\mathbf{r}}_i \tag{20}$$

The $\tilde{\mathbf{c}}_i$ which minimizes the MSE $E[(b_{i,1} - \tilde{\mathbf{c}}'_i \tilde{\mathbf{r}}_i)^2]$ is

$$\tilde{\mathbf{c}}_{mmse} = \tilde{\mathbf{R}}^{-1} \tilde{\mathbf{p}} \tag{21}$$

where

$$\tilde{\mathbf{R}} = E(\tilde{\mathbf{r}}_i \tilde{\mathbf{r}}'_i) = \mathbf{S}'_D \mathbf{R} \mathbf{S}_D \tag{22}$$

and

$$\tilde{\mathbf{p}} = \mathbf{S}'_D E(b_{i,1}\mathbf{r}_i) = \mathbf{S}'_D \mathbf{s}_1 \tag{23}$$

The $\tilde{\mathbf{c}}_i$ which minimizes the LS cost function (11) (where $\mathbf{c}$ and $\mathbf{r}$ are replaced by $\tilde{\mathbf{c}}$ and $\tilde{\mathbf{r}}$, respectively) is

$$\tilde{\mathbf{c}}_i = \hat{\tilde{\mathbf{R}}}_i^{-1} \hat{\tilde{\mathbf{p}}}_i \tag{24}$$

where

$$\hat{\tilde{\mathbf{R}}}_i = \sum_{i=i-B+1}^{i} \tilde{\mathbf{r}}_i \tilde{\mathbf{r}}'_i = \mathbf{S}'_D \hat{\mathbf{R}}_i \mathbf{S}_D \tag{25}$$

and

$$\hat{\tilde{\mathbf{p}}}_i = \sum_{i=i-B+1}^{i} \hat{b}_{i,1} \tilde{\mathbf{r}}_i = \mathbf{S}'_D \hat{\mathbf{p}}_i \tag{26}$$

The optimal subspace $\mathbf{S}_D$ with respect to the LS criterion was derived in [20]. Since $\hat{\mathbf{R}}_i$ is symmetric and positive semi-definite, we can write

$$\hat{\mathbf{R}}_i = \Phi \Lambda \Phi' \tag{27}$$

where the columns of Φ are the orthonormal eigenvectors of $\hat{\mathbf{R}}_i$, and Λ is the diagonal matrix of eigenvalues. Choosing $\mathbf{c}_i$ to minimize the LS cost function C_{LS} gives

$$C_{LS} = 1 - \hat{\mathbf{p}}'_i \hat{\mathbf{R}}_i \hat{\mathbf{p}}_i = 1 - \mathbf{q}'_i \Lambda^{-1} \mathbf{q}_i = 1 - \|\Lambda^{-1}\mathbf{q}_i\|^2 \tag{28}$$

where

$$\mathbf{q}_i = \Phi' \hat{\mathbf{p}}_i \tag{29}$$

We therefore conclude that the subspace which minimizes the cost function C_{LS}, assuming an RRLS solution of dimension D, has basis vectors which are the eigenvectors of $\hat{\mathbf{R}}_i$ associated with the D largest values of $|\mathbf{q}_{i,k}/\Lambda_{k,k}|^2$, where $\mathbf{q}_{i,k}$ is the kth component of $\mathbf{q}_i$. That is, the optimal projection matrix $\mathbf{S}_D$ has columns consisting of this set of eigenvectors.

Determination of the optimal subspace (specifically, $\mathbf{q}_i$) for the RRLS solution depends on the decisions $\hat{b}_{i,1}$, $i = i - B + 1, \ldots, i$. This presents a problem, analogous to generating decisions for the decision-directed BLS algorithm. An alternative is to choose as a basis for the subspace the D eigenvectors of $\hat{\mathbf{R}}_i$ corresponding to the D largest eigenvalues. The projection in that case is onto the subspace spanned by the *strongest* signal components, also referred to as "principal components". The motivation for the projection is that the MMSE filter $\mathbf{c}_i$ lies within the signal subspace. Projecting $\mathbf{c}_i$ onto the signal subspace therefore incurs no loss in performance. Although suboptimal in the LS sense, the principal components projection avoids the problem of having to first estimate the symbols over the block of interest in order to choose the subspace. This type of subspace filtering has been used extensively in array processing [18], [17], [19]. Reduced-rank filtering for interference suppression in DS-CDMA is also studied in [21], where a subspace tracking algorithm is used to estimate the signal subspace, in contrast to the batch-oriented eigen-decomposition used here. Signal subspace techniques have also been proposed for timing and channel estimation in DS-CDMA [28], [29], [30].

4 Numerical Results

4.1 Performance Comparison

Figures 1 and 2 show plots of (uncoded) error rates as a function of traffic load (measured in Erlangs/cell) for the model and detection algorithms discussed in the preceding two sections. The traffic load is λ/μ where λ is the packet arrival rate per cell, and $1/\mu$ is the average number of symbols per packet. The mean packet length is fixed at $1/\mu = 1000$, so that the traffic load is varied by changing λ. The processing gain $N = 32$ and the background signal-to-noise ratio $A_1^2/\sigma^2 = 18$ dB. All results assume a cluster of seven cells, so that the other-cell traffic load is six times the intra-cell traffic load. In Figure 1, the standard deviation for the power of the intra-cell users is 1 dB (tight power control), and in Figure 2 this standard deviation is 6 dB (loose power control). Most points were obtained by simulating the model for 400 different packets (4×10^5 iterations on average). At higher traffic loads, fewer packets are needed to obtain statistically significant results.

Figures 1 and 2 show error rates vs. traffic load for the decision-directed RLS, decision-directed BLS, and Orthogonally-Anchored BLS (OALS) algorithms. These are compared with both the matched filter receiver, and with the MMSE (N-tap) receiver. Although the LMS algorithm was simulated, it was unable to track the interference transients, and generally performed much worse than the matched filter. Consequently, those results have been omitted from Figures 1 and 2. The error rate of each decision-directed algorithm was monitored over a window size of 400 symbols. If the error rate exceeded 15%, the algorithm switched to blind mode. This switch is especially important at high traffic loads, and for the decision-directed BLS algorithm, which has a tendency to become unstable at moderate to high error rates.

At moderate traffic loads, the total number of users in the system may be quite large relative to the processing gain. For example, at 10 Erlangs/cell, there are an average of 70 asynchronous users in the system, which far exceeds the processing gain. A zero-forcing (decorrelating) solution for $\mathbf{c}$ does not exist in this situation, yet the adaptive receivers still offer a significant improvement in performance relative to the matched filter. For example, at an error rate of 5%, Figure 1 shows that the RLS algorithm can support nearly twice the traffic load as the matched filter. Figures 1 and 2 also show that the performance of the adaptive algorithms is insensitive to power variations across the user population. The gain in capacity provided by the adaptive receivers therefore increases as power control is relaxed.

As the traffic load increases, Figures 1 and 2 show that the performance advantage of the adaptive receivers relative to the matched filter diminishes. This is due to the finite number of dimensions available with which it can suppress interference. A heuristic interpretation is that with N available dimensions (tap coefficients), the filter is only able to suppress the N strongest interferers down to the level of the next strongest interferer. As the number of strong interferers increases, the resulting improvement in performance diminishes.

The performance of each adaptive algorithm is affected significantly by the expo-

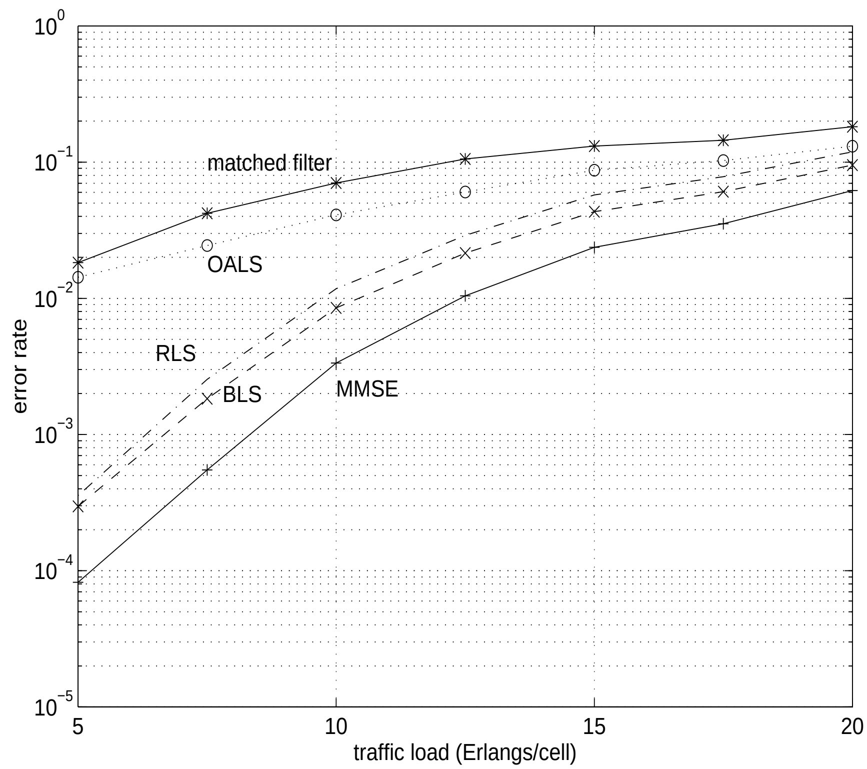

Figure 1: Uncoded error rate vs. traffic load, measured in Erlangs per cell, for the matched filter ($- * -$), RLS algorithm ($-\cdot$), BLS algorithm ($- \times -$), blind (OALS) algorithm ($-o-$), and MMSE solution ($- + -$). The received average power for the intra-cell users has standard deviation 1 dB. ($N = 32$, mean packet length$= 1000$, block length for BLS algorithm$= 400$)

nential weight or block length (whichever is relevant). The results in Figures 1 and 2 were generated after some experimentation with these parameters. The block length chosen for the BLS algorithms was 400 symbols, and the exponential weight for the RLS algorithms was 0.995. Optimal selection of these parameters in general depends on the traffic load and processing gain.

The Figures show that the blind LS algorithms do not perform as well as the decision-directed algorithms. Consequently, the main utility of the blind algorithms is avoiding instability due to unreliable decisions (e.g., due to the appearance of a new user). This is quite important, especially for the BLS algorithm at moderate error rates. The difference in performance between the blind and decision-directed algorithms depends on the block length (or exponential weight). As the block length increases, or the exponential weight approaches one, the performance of the orthogonally-anchored algorithm approaches that of the decision-directed algorithm, assuming no mismatch. That is, we assume that the blind algorithms have accurate estimates of the received signal for the desired user, which may pose a problem when multipath is present (or if timing offset is present, as discussed in Section

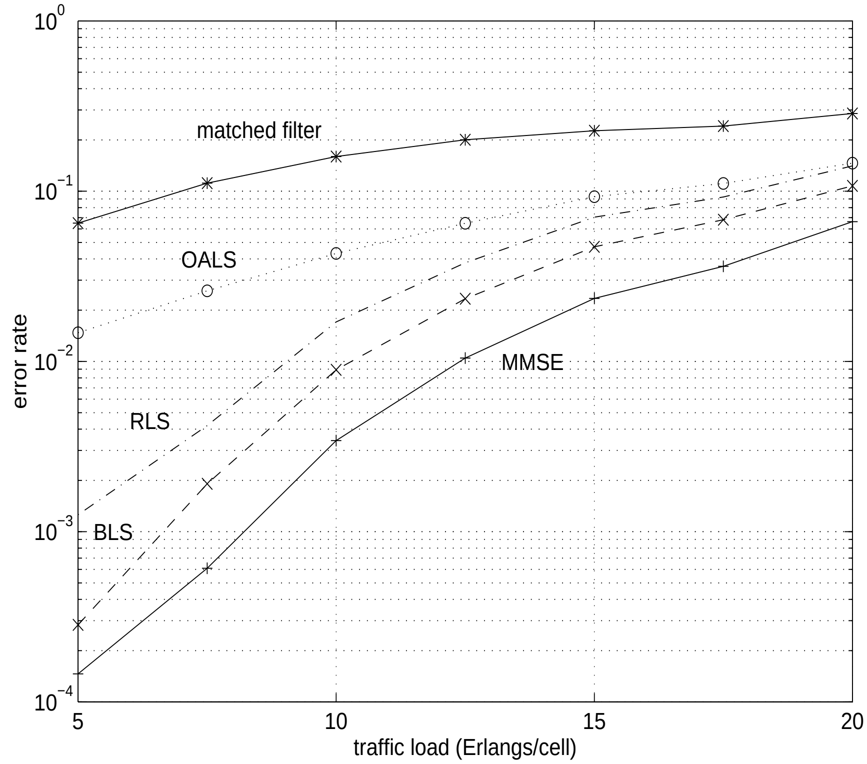

Figure 2: Uncoded error rates vs. traffic load. The received average power for the intra-cell users has standard deviation 6 dB. All other parameters are the same as those in Figure 1.

4.3).

4.2 RRLS Results

Figure 3 illustrates the performance of the RRLS technique. Error rate is shown as a function of the dimension of the filter $\tilde{\mathbf{c}}$ (that is, the subspace dimension). The traffic load is 5 Erlangs/cell, so that the dimension of the space spanned by the intra-cell users (10 on average) is much less than the processing gain. Two sets of results are shown corresponding to different block lengths over which the filter is computed. For the longer block length (400) the performance degrades as the dimension of the receiver filter decreases, whereas for the shorter block length (100) the optimum subspace dimension is approximately 20, and the choice of dimension has a significant affect on performance.

These results indicate that the RRLS technique is beneficial for short blocks. Figure 3 also shows that significant performance degradation is incurred by choosing the short block length relative to the longer block length. This is not surprising since short block lengths create problems with data sufficiency. Namely, when the block length is short relative to the number of filter coefficients, there is insufficient data to obtain a good estimate of the optimal filter. For the CDMA application considered, we observe that to perform close to optimal (with respect to MSE), the block length should be at least 10 times the number of filter coefficients. For short blocks RRLS can improve performance since it reduces the number of coefficients to be estimated.

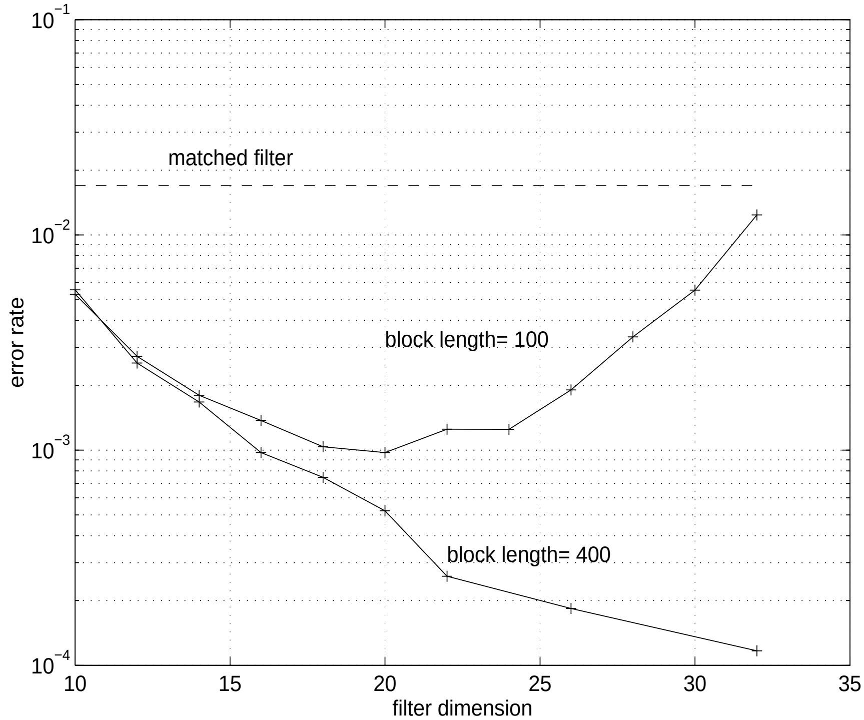

Figure 3: Uncoded error rate vs. dimension of receiver filter ($\tilde{\mathbf{c}}$) for the RRLS algorithm. Results for two block lengths, 100 and 400, are shown. The traffic load is 5 Erlangs/cell, and all other parameters are the same as those in Figure 1.

4.3 Timing Offset

Figure 4 shows error rates as a function of timing offset, normalized by the chip duration, for the matched filter, adaptive receivers, and MMSE receiver. The traffic load is fixed at 10 Erlangs per cell. As mentioned in Section 3.3, the OALS algorithm requires the modification (18). To generate the results in Figures 4 and 5, the constant δ was taken to be 0.5. (This value was chosen after some experimentation). All receivers show a graceful degradation in error rate as the timing offset increases. For small timing offsets ($< 10\%$) the degradation in performance is relatively minor for all receivers.

Figure 5 shows error rates vs. traffic load for the matched filter and adaptive receivers with a 20% timing offset. The standard deviation for the received intra-cell power is 1 dB. Comparing these results to those in Figure 1, we see that the improvement in performance of the adaptive algorithms relative to the matched filter is not significantly affected by timing offset. This robustness with respect to timing offset is an advantage of the adaptive filtering approach relative to other implementations of linear decorrelating and MMSE receivers [22], [23]. We also mention related work in which adaptive algorithms are used to acquire timing (in addition to demodulation) [31],[32].

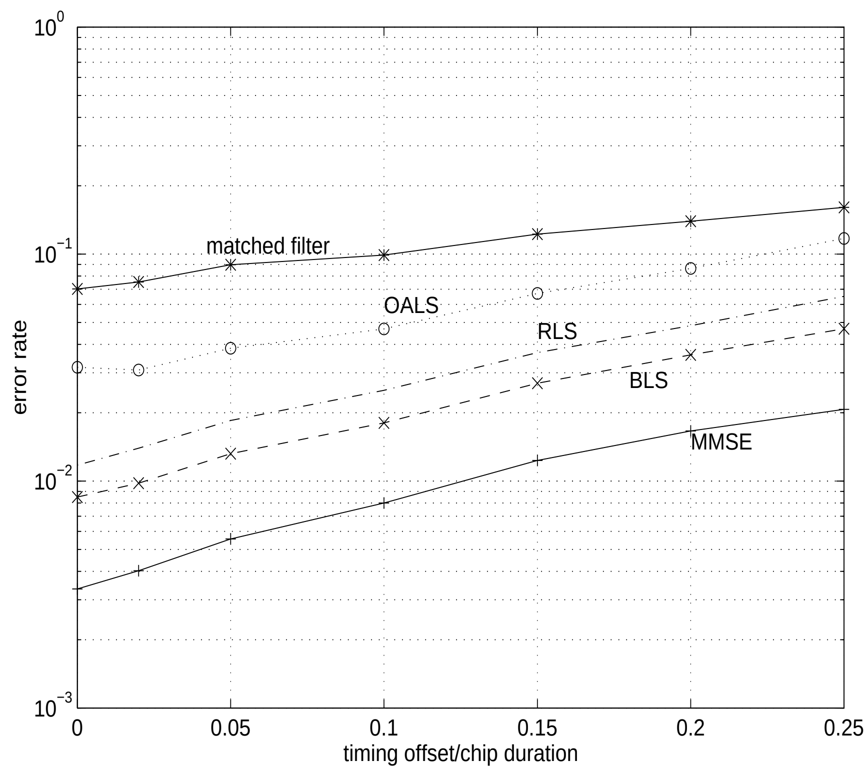

Figure 4: Uncoded error rate vs. timing offset, normalized by the chip duration, for the matched filter $(-*-)$, RLS algorithm $(-\cdot)$, BLS algorithm $(-\times-)$, blind (OALS) algorithm $(-o-)$, and MMSE solution $(-+-)$. The traffic load is 10 Erlangs/cell, and all other parameters are the same as those in Figure 1.

5 Conclusions

The main contribution of this paper, relative to previous work on adaptive interference suppression for DS-CDMA, is to introduce a model for packet data transmission which produces transients in the interference environment, and to present associated performance results. The results in the preceding section indicate that for the model considered, a significant increase in traffic load can be supported with adaptive LS detectors, relative to the matched-filter detector. This capacity increase diminishes

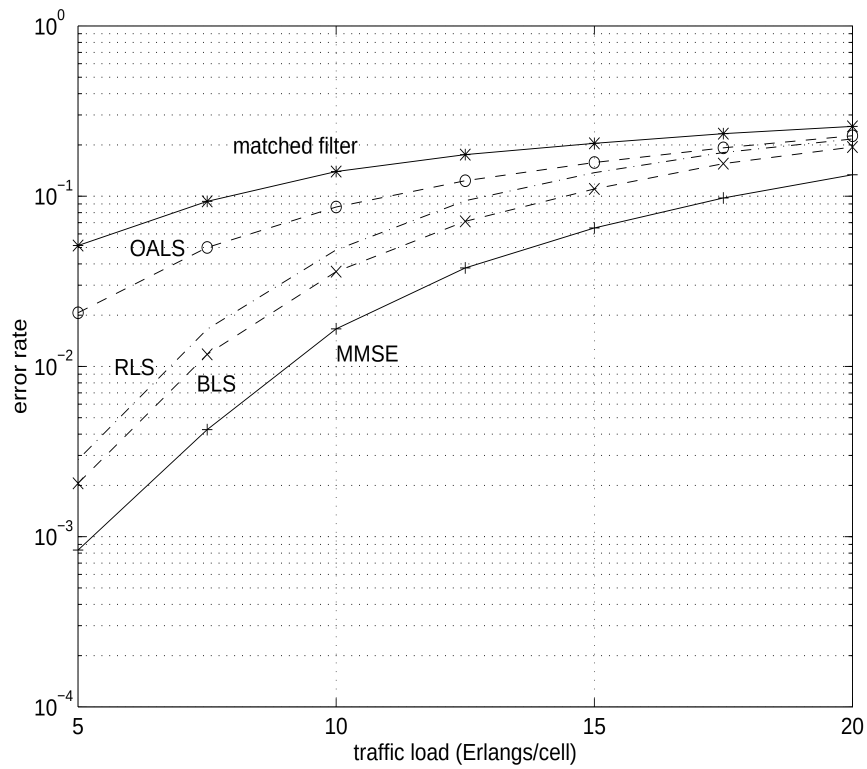

Figure 5: Uncoded error rates vs. traffic load with a normalized timing offset of 20%. All other parameters are the same as those in Figure 1.

as the traffic load increases (i.e., becomes a large fraction of the processing gain). The performance of the adaptive algorithms is insensitive to variations in received power over the user population, which can alleviate power control requirements. Of course, the primary disadvantage of the adaptive algorithms is additional signal processing complexity.

The results for the RRLS algorithm show that it offers a performance improvement only for relatively short block lengths. For the model considered, short block lengths cause a significant degradation in performance due to data insufficiency. In general, the RRLS approach should be useful when the number of filter coefficients is much larger than the dimension of the signal space. This may be the case when multiple antennas are combined with time-domain interference suppression.

There are many directions in which this work can be extended. A signal processing enhancement to conventional adaptive algorithms which attempts to detect and rapidly suppress each new user is described in [33], and may provide some performance improvement relative to conventional LS algorithms. Inclusion of flat Rayleigh fading in the packet data model described here, along with associated performance results, is described in [14]. Future work will incorporate frequency-selective fading, as well as system enhancements, such as coding, interleaving and diversity, into the model.

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