Adaptive Techniques for

Multiuser CDMA Receivers

Enhanced Signal Processing with Short Spreading Codes

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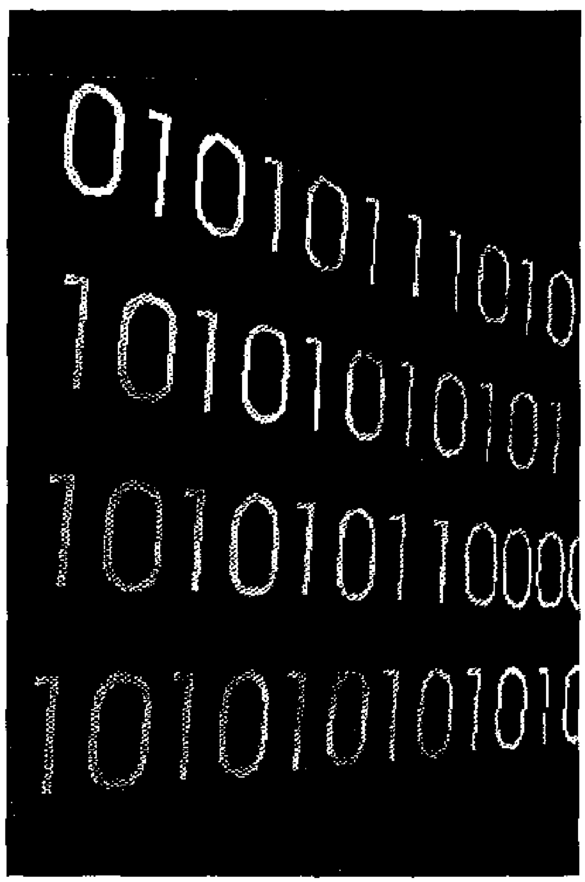

Spread-spectrum systems have a long history in military and civilian wireless communications [1]. As originally conceived, they did not involve elaborate signal processing, nor were they envisioned as a way of arbitrating channel resources among multiple users. The merits of spread-spectrum modulation for multiplexing voice users (code division multiple access—CDMA) are now widely accepted for cellular applications [2]. Still, existing CDMA systems (like IS-95) include limited signal processing and interference suppression, namely, the single-user matched filter or RAKE receiver, which treats interference from other users as noise. The statistical averaging of out-of-cell interference and exploitation of silence periods in voice conversations made possible in the CDMA environment provide unique benefits for cellular applications compared with rival TDMA/FDMA options. For this reason, most proposals considered for third-generation wireless networks involve some flavor of CDMA [3]. It is expected that the requirements imposed on third-generation systems in terms of capacity and flexibility will necessitate advanced signal processing solutions for interference suppression and joint decoding of multiple users.

It was observed in the mid-1980s that joint, optimal, maximum-likelihood decoding of all users has significant performance benefits compared with matched filter alternatives [4]. Unfortunately, the solution also involves a joint Viterbi processor with exponential complexity in the number of users. The seminal work of [4] and the promised gains of multiuser detection (MD) have initiated much research in the area which continues unabated to this day. A number of CDMA receivers have been proposed that cover the whole spectrum of perfor-

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mance/complexity from the simple matched filter to the optimal Viterbi processor. Adaptive solutions, in particular, have the potential of providing the anticipated MD performance gains with a complexity that would be manageable for third generation systems.

Our goal, in this article, is to provide an overview of recent work in MD with an emphasis on adaptive methods. We start with (suboptimal) linear receivers and discuss the data-aided MMSE receiver. Blind (nondata-aided) implementations are also reviewed together with techniques that can mitigate possible multipath effects and channel dispersion. In anticipation of those developments, appropriate discrete-time (chip rate) CDMA models are reviewed, which incorporate asynchronism anc channel dispersion.

For systems with large spreading factors, the convergence and tracking properties of conventional adaptive filters may be inadequate due to the large number of coefficients which must be estimated. In this context, reduced rank adaptive filtering is discussed. In this approach, the number of parameters is reduced by restricting the receiver tap vector to belong to a carefully chosen subspace. In this way the number of coefficients to be estimated is significantly reduced with minimal performance loss.

It is well known in the context of single-user channel equalization that decision feedback (DF) adaptive schemes can provide near-optimal performance with little added complexity. This is true also in the case of multiuser systems; the difference here is that interference does not only originate from past symbols, but also from current symbols of other (interfering) users. Therefore, tentative decisions for the interfering symbols are needed to implement such interference cancellation schemes providing further justification for the use of linear receivers (as a preprocessing step). Both adaptive sequential and parallel decision-feedback strategies are possible as explained later. These receivers are motivated by a brief discussion of fundamental limits on performance (capacity) with error control coding.

The receiver design is affected by the type of spreading sequences used. This article deals with short spreading codes which repeat every symbol period. Some systems (like IS-95) employ long codes (with period much longer than the symbol period) which causes the interference to vary randomly from symbol to symbol [5]. Usually, systems which employ long codes also employ large spreading factors and a large number of users per CDMA channel, in an effort to randomize the interference further and justify the use of the matched filter receiver. An alternative approach is to combine short codes with the adaptive receivers discussed here, which exploit the structure of the interference. The use of relatively small spreading factors combined with a proportionately smaller number of users reduces complexity and facilitates the use of advanced signal processing. Of course smaller spreading factors also imply smaller bandwidth expansion, so that the number of users accommodated per Hertz is not reduced.

A linearly modulated digital communications signal is cyclostationary with period equal to the symbol period. CDMA signals with short spreading sequences fall into this category. For systems with long codes, the chip-sampled interference is stationary, if the codes are unkown. If the interferers' codes are known, then the interference can be modeled as a time-varying, cyclostationary process. In general, long codes complicate the development of adaptive signal processing algorithms for multiuser detection [6]-[11] and will not be treated here.

CDMA Signal Model

In CDMA systems all users transmit simultaneously in the same frequency band. Therefore if K users are active, the received, baseband, continuous-time signal is a superposition of all K signals

$$r(t)=\sum_{k=1}^{K} r_k(t)+n(t) \tag{1}$$

where $n(t)$ is additive Gaussian noise and and each user's signal is

$$r_k(t)=\sum_i A_k b_k[i] p_k(t-iT_s-\nu_k) \tag{2}$$

a superposition of signature waveforms $p_k(t)$ spaced by multiples of the symbol period T_s and linearly modulated by the information symbol sequence $b_k[i]$ with amplitudes A_k. In the case of asynchronous systems, each user may have a different delay ν_k.

It is desirable to utilize different signature waveforms for different users (with sufficient excess bandwidth) to facilitate signal separation at the receiver. Often each user's signature is generated by modulating its low rate symbol waveform with a high rate code waveform (see Fig. 1). While the description of Fig. 1 is conceptually simple, it does not lend itself to the development of appropriate discrete-time baseband models useful in receiver design. In an effort to derive chip-rate models for CDMA systems, we may write the spread-spectrum signature as a succession of chip pulses $h(t)$

$$p_k(t)=\sum_{l=0}^{N-1} c_k[l]h(t-lT_c) \tag{3}$$

modulated by the user code $c_k[l]$, $k=0,\ldots,N-1$, where N is the number of chips per bit (or processing gain) and T_c is the chip period. In the case of propagation in a dispersive environment, the waveform $h(t)$ represents the convolution of the chip pulse with the channel response. If the received signal is sampled at a fraction of the chip period (P samples per chip), then the system is described by the following multirate convolution:

$$r[n]=r(t)\big|_{t=n\frac{T_c}{P}}=\sum_{k=1}^{K}\sum_i A_k b_k[i]p_k[n-iNP] \tag{4}$$

where

$$p_k[n]=p_k(t)\big|_{t=n\frac{T_c}{P}}=\sum_{l=0}^{N-1} c_k[l]h[n-lP] \tag{5}$$

where $h[n]$ is a similarly fractionally sampled version of $h(t-\nu_k)$. Fig. 2 gives a multirate interpretation of (4) and (5). The spectral spreading operation may be considered as an upsampling of the information symbol sequence by N followed by a finite impulse response (FIR) filter of length N with impulse response equal to the spreading code. Then, the chip-fractionally sampled channel model is given by one more multirate structure with upsampling factor P. The model of Fig. 2 is a most general description of the received signal and makes no assumptions on the channel spread or on the amount of possible intersymbol interference (ISI). In that respect, it is useful in describing CDMA systems with potentially small processing gain and large channel spread like the one considered for the UMTS third-generation wireless systems [12].

Vector models of the received data (within a certain observation window) are more desirable and can be derived from Fig. 2 following standard multirate tools (e.g., polyphase decomposition [13, Ch. 4]). For example, in the case of synchronous users with $P=1$ (one sample per chip), and no interchip interference (ICI) the user's signature has length N, $\mathbf{p}_k=[p_k[0],\ldots,p_k[N-1]]^T$ and is a scalar multiple of the user code $\mathbf{p}_k=A_k\mathbf{c}_k=A_k[c_k[0],\ldots,c_k[N-1]]^T$. Then, the received signal can be written in matrix form as

$$\mathbf{r}[n]=\mathbf{P}\mathbf{b}[n]+\mathbf{n}[n] \tag{6}$$

where $\mathbf{P}=[\mathbf{p}_1,\ldots,\mathbf{p}_K]$ and $\mathbf{b}[n]=[b_1(n),\ldots,b_K(n)]^T$. If ICI is present due to a dispersive channel of order q, then the user signature has length $N+q$ and is given by the convolution of the code $c_k[i]$ with the channel response $h_k[i]$. Therefore, the vector $\mathbf{p}_k=\mathcal{T}(\mathbf{c}_k)\mathbf{h}_k$ is given by the channel vector $\mathbf{h}_k$ multiplied by the Toeplitz filtering matrix constructed from the code $\mathbf{c}_k$

$$\mathcal{T}(\mathbf{c}_k)=\begin{bmatrix} c_1(0) & & 0 \\ \vdots & \ddots & c_1(0) \\ c_1(N-1) & & \vdots \\ 0 & \ddots & c_1(N-1)\end{bmatrix}.$$

If the length of the vector $\mathbf{p}_k$ is greater than N, then this leads to ISI.

The length of the observation window depends on the choice of system parameters. If $q \ll N$ and ISI is negligible, then $\mathbf{r}[n]$ has length N. In the case of asynchronism and/or severe ISI, however, a longer window should be considered that spans multiple symbols.

Minimum Mean Squared (MMSE) Linear Receivers

Given the received vector of samples at the output of the chip matched filter for symbol i, a linear multiuser detector forms the (soft) estimate

$$\tilde{\mathbf{b}}[i]=\mathbf{W}^H\mathbf{r}[i] \tag{7}$$

where $\mathbf{W}$ is an $N\times K$ matrix. (Here we assume the filter spans a single symbol interval.) We can select $\mathbf{W}$ to minimize mean squared error (MSE), defined as

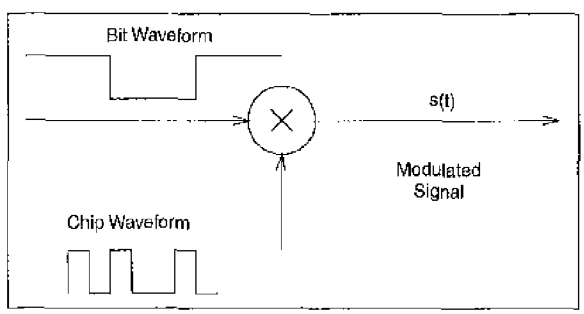
$$\xi=E\left\{\|\mathbf{b}[i]-\tilde{\mathbf{b}}[i]\|^2\right\}. \tag{8}$$

The solution is given by

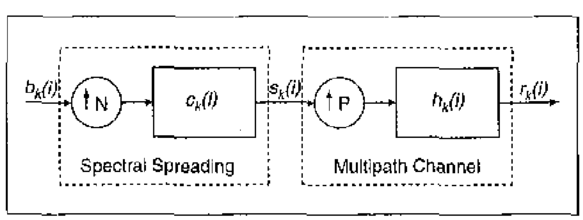
$$\mathbf{W}=\mathbf{P}\left(\mathbf{P}^H\mathbf{P}+\sigma^2\mathbf{I}\right)^{-1} \tag{9}$$

where $E(\mathbf{n}[i]\mathbf{n}^H[i])=\sigma^2\mathbf{I}$.

The linear MMSE receiver has the important property that a single user can be detected without having to detect



▲ 1. *Spectral spreading: continuous-time model.*



▲ 2. *DS/SS signal in multipath: discrete-time model.*

all other users. That is, the kth column of $\mathbf{W}$, which is used to detect user k, is given by

$$\mathbf{w}_k = \mathbf{R}^{-1}\mathbf{p}_k \tag{10}$$

where $\mathbf{R} = E\{\mathbf{r}[i]\mathbf{r}^+[i]\}$ is the input covariance matrix. In other words, linear multiuser detection can be implemented as a set of "single-user" interference suppression filters, and is therefore well suited for the DS-CDMA forward link (in addition to the reverse link).

Some important properties of linear MMSE detection are:

▲ The filter for each user can be implemented as an adaptive digital FIR filter, analogous to an adaptive equalizer for a single-user channel.

▲ When implemented as an adaptive filter, the linear MMSE detector suppresses *total* interference, independent of origin. It therefore suppresses (strong) *other-cell* interference, in addition to intra-cell interference.

▲ When implemented as an adaptive filter, the MMSE receiver requires little side information. Specifically, either a training sequence is needed at the start of each transmission or the receiver must know the desired user's spreading code, channel, and associated timing. Amplitudes, phases, and spreading codes of interferers are not required for adaptation.

▲ In principle, the filter can suppress $N-1$ interferers for synchronous CDMA. For asynchronous CDMA, a digital filter that spans a single symbol interval can suppress $\lfloor (N-1)/2 \rfloor$ interferers. By increasing the observation window, the filter can suppress up to $N-1$ users [14], [15]; however, adaptation becomes more difficult.

▲ The (coherent) MMSE solution automatically combines all multipath within the window spanned by the filter.

▲ The performance of the linear MMSE receiver degrades gracefully with the number of (equal power) users (e.g., see [16]), although for very large loads $K/N >> 1$ (K= number of strong users, N= processing gain), the performance of the MMSE receiver is close to that of the matched filter.

Blind Minimum Output Energy Methods

MMSE solutions are typically implemented with the aid of training sequences. Even in the absence of training data, however, (10) indicates that the solution is implementable if the signature and timing of the user of interest is known. In particular, the covariance matrix needed in (10) can be estimated from the data, while (in the absence of ICI) the user's signature coincides with the spreading code and may be readily available. This problem is analogous to beamforming problems appearing in array processing, in which the direction of arrival and signature of the user of interest are known (e.g., [17]). Adaptive implementation of such receiver filters which explicitly take into account the signature of the user of interest can be developed using constrained optimization approaches [18].

In the context of CDMA and in the absence of dispersive channels, an MMSE receiver can be obtained by minimizing the output energy $E\{|\tilde{b}_1[i]|^2\}$ (let user 1 be the user of interest without loss of generality) [19]. To avoid the trivial solution $\mathbf{w}_1 = 0$, the response of the receiver to the user of interest is constrained to one $\mathbf{w}_1^H\mathbf{p}_1 = 1$. This constrained optimization problem has been studied extensively in the context of array processing and the solution is termed minimum variance distortionless beamformer (e.g., [17]). Alternatively, the adaptive filter can be decomposed as $\mathbf{w}_1 = \mathbf{p}_1 + \mathbf{x}_1$ where $\mathbf{x}_1$ is adapted but is always forced to be orthogonal to the signature $\mathbf{p}_1$ [19]. Extensions to longer observation intervals and separate treatment of the in-phase and quadrature signals were presented in [20]. Similar approaches in array processing come under generalized sidelobe cancellers. The solution can be shown to be a scalar multiple of the MMSE solution and the minimum output energy (MOE) is

$$MOE(\mathbf{p}_1) = \frac{1}{\mathbf{p}_1^H\mathbf{R}^{-1}\mathbf{p}_1}. \tag{11}$$

Unfortunately, it was observed in [19], and in prior array processing literature, that those constrained optimization approaches are very sensitive to possible signature mismatch created by multipath effects or timing errors. If the actual user signature differs from the one assumed in the derivation of the receiver, significant signal cancellation can occur resulting in poor performance. In [21] and [22], the problem is mitigated by constraining the solution to the signal subspace, to reduce signal cancellation. An adaptive implementation based on subspace tracking was shown to improve performance at the expense of more computational complexity.

The method of [19] was later extended by adding more constraints [23]. In particular, a solution for the dispersive channel case was attempted in [24] and later in [25], by forcing the receiver response to delayed copies of the signal of interest to zero. Given the structure of the user's signature in multipath $\mathbf{p}_1 = \mathcal{T}(\mathbf{c}_1)\mathbf{h}_1$, the receiver vector is constrained to satisfy $\mathbf{w}_1^H\mathcal{T}(\mathbf{c}_1) = [0,\ldots,1,\ldots,0]$. With these additional constraints, minimum variance techniques are applicable, but have inferior performance since they treat part of the useful signal as interference.

This obstacle was overcome by constrained optimization solutions which combine all multipath components of the signal of interest and jointly minimize the interference, while maximizing the signal component at the receiver's output [26], [27]. The idea is again borrowed from array processing, known by the term Capon beamformer [28], [17]. If the user signature is parameterized by some unknown parameters, then the Capon solution selects those parameter values which maximize the minimum output energy. In array process-

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ing those parameters typically refer to directions of arrival, while in CDMA to channel tap values. Based on (11), the Capon receiver selects $\hat{\mathbf{h}}_1$ such that

$$\hat{\mathbf{h}}_1 = \arg\max_{\mathbf{h}} \frac{\mathbf{h}^H \mathbf{h}}{\mathbf{h}^H \mathcal{T}^H(\mathbf{c}_1)\mathbf{R}^{-1}\mathcal{T}(\mathbf{c}_1)\mathbf{h}}. \quad (12)$$

The rational for this minimax optimization setup relates to an effort to maximize the signal component in the output after the interference has beeen suppressed. Equation (12) represents a Rayleigh quotient and hence the solution corresponds to the principal eigenvector of the matrix $\mathcal{T}^H(\mathbf{c}_1)\mathbf{R}^{-1}\mathcal{T}(\mathbf{c}_1)$. Finally, given $\hat{\mathbf{h}}_1$, the receiver vector can be obtained as a multiple of the MMSE solution $\mathbf{R}^{-1}\mathcal{T}(\mathbf{c}_1)\hat{\mathbf{h}}_1$.

Those Capon blind methods exhibit superior performance which under some circumstances is close to that of the trained MMSE receiver [26]. Futhermore, at high SNR $\hat{\mathbf{h}}_1$ can be shown to converge to the true channel parameters $\mathbf{h}_1$. Figure 3 illustrates these claims for a system with ten users, spreading factor $N = 31$ and a severe near-far effect. Two variations of the Capon method are compared with the trained MMSE detector and show close signal-to-interference-plus-noise-ratio (SINR) performance (for details see [26]). On the other hand, if the MOE method is applied with no regard for the multipath induced signature distortion, a substantial SINR penalty is incurred.

Adaptive implementations are proposed in [29] through two coupled least mean square (LMS) recursions, jointly updating $\mathbf{w}_1$ and $\mathbf{h}_1$. Blind solutions with performance which is identical to that of the trained MMSE receiver in a dispersive environment are possible as was demonstrated in [30] and [31]. Those techniques make explicit use of subspace information from the autocorrelation matrix and do not lend themselves to time-recursive implementations. Related developments also include [32], [9], and [33].

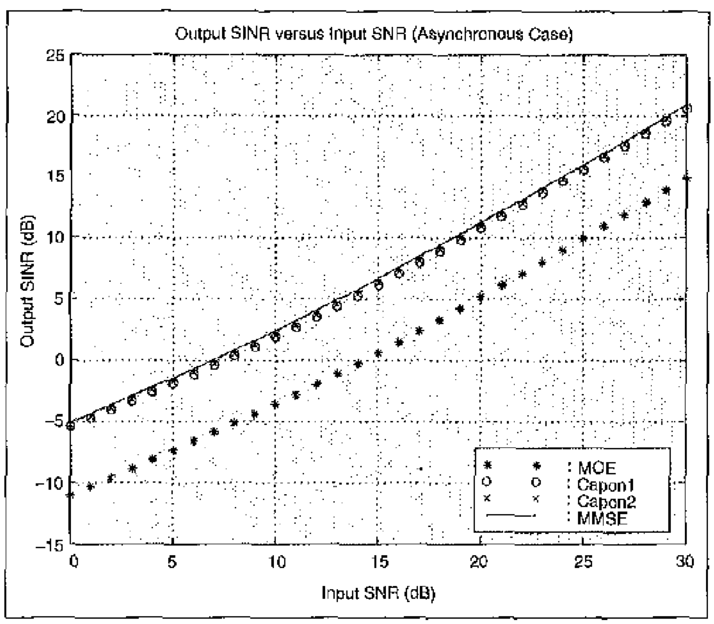
Reduced-Rank Approximations

Given an adequate number of data samples, the algorithms presented in the preceding section simultaneously suppress multiple access interference and perform multipath combining. Tracking and convergence may be problems, however, for some wireless systems in which a large number of filter coefficients must be estimated. For example, a conventional implementation of a time-domain adaptive filter which spans three symbols for proposed third-generation wideband CDMA cellular systems can have over 300 coefficients. Introducing multiple antennas for additional space-time interference suppression capability exacerbates this problem. Adapting such a large number of filter coefficients implies very slow response to changing interference and channel conditions.

A reduced-rank filter first projects the received signal onto a lower-dimensional subspace before processing. This type of dimension reduction can improve tracking and convergence in time-varying environments. Reduced-rank linear filtering has been studied primarily for array processing and radar applications (e.g., see [17], [34]); however, recently it has been proposed for interference suppression in direct-sequence (DS)-CDMA systems [35]-[38], [21].

In what follows for simplicity we assume a synchronous CDMA channel without multipath. The generalization to asynchronous CDMA is straightforward, where the filter may span multiple symbols. Although there has been some work in applying reduced-rank techniques to frequency-selective channels [21], this is currently an active area of research.

Let $\mathbf{M}_D$ be the $N \times D$ matrix with column vectors which are an orthonormal basis for a D-dimensional



▲ *3. Performance of blind algorithms for an asynchronous system.*

subspace, where $D < N$. The projected received vector corresponding to symbol i is then given by

$$\tilde{\mathbf{r}}[i] = \mathbf{M}_D^H \mathbf{r}[i]. \tag{13}$$

The sequence of projected received vectors $\{\tilde{\mathbf{r}}[i]\}$ is the input to a tapped-delay line filter, represented by the D-vector $\tilde{\mathbf{w}}[i]$ for symbol i. The filter output corresponding to the ith transmitted symbol is $z[i] = \tilde{\mathbf{w}}^H[i]\tilde{\mathbf{r}}[i]$, and the objective is to select $\tilde{\mathbf{w}}$ to minimize the reduced-rank MSE

$$\mathcal{M}_D = E\left\{|b_1[i] - \tilde{\mathbf{w}}^H \tilde{\mathbf{r}}[i]|^2\right\}. \tag{14}$$

The solution is

$$\tilde{\mathbf{w}} = \tilde{\mathbf{R}}^{-1}\tilde{\mathbf{c}}_1 \tag{15}$$

where

$$\tilde{\mathbf{R}} = \mathbf{M}_D^H \mathbf{R} \mathbf{M}_D \tag{16}$$

$$\tilde{\mathbf{c}}_1 = \mathbf{M}_D^H \mathbf{c}_1. \tag{17}$$

In what follows we describe a few different reduced-rank techniques which have been considered. Other related reduced-rank methods have been proposed in [35] and [38]-[40].

Methods Based on Eigen-Decomposition

The reduced-rank technique which has probably received the most attention is "principal components (PC)," which is based on the following eigen-decomposition of the covariance matrix

$$\mathbf{R} = \mathbf{V}\Lambda\mathbf{V}^H \tag{18}$$

where $\mathbf{V}$ is the orthonormal matrix of eigenvectors of $\mathbf{R}$ and Λ is the diagonal matrix of eigenvalues. Given this decomposition, the received vector $\mathbf{r}$ is then projected onto the D-dimensional subspace which contains the most energy. Suppose that the eigenvalues are ordered as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$. For given subspace dimension D, the projection matrix for PC is then the first D columns of $\mathbf{V}$.

For $K < N$, the eigenvalues $\lambda_1, \ldots, \lambda_K$ are associated with the signal subspace, and the remaining eigenvalues are associated with the noise subspace, i.e., $\lambda_m = \sigma^2$ for $K < m < N$. Consequently, by selecting $D \geq K$, PC retains full-rank MMSE performance (e.g., see [21] and [41]). However, the performance can degrade quite rapidly for $D \leq K$, since there is no guarantee that the associated subspace will retain most of the desired signal energy. This is especially troublesome in a near-far scenario, since for small D, the subspace which contains most of the energy will likely correspond to the interference, and not the desired signal. We remark that in a heavily loaded cellular system, the dimension of the signal subspace may be near, or even exceed the number of dimensions available, in which case PC does not offer much of an advantage relative to conventional full-rank adaptive techniques.

An alternative to PC is to choose a set of D eigenvectors for the projection matrix which minimizes the MSE. Specifically, assuming that the variance of the data symbols is one, we can write the full-rank MSE in terms of projected variables as

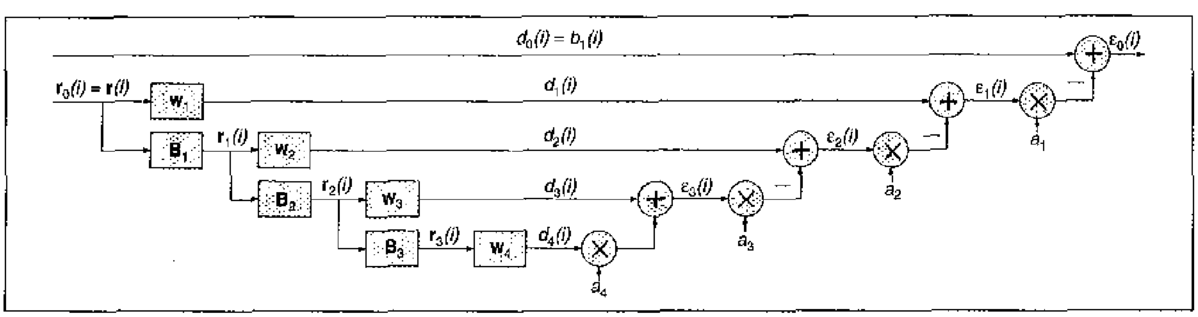
$$\mathcal{M} = 1 - \left\|\Lambda^{-1}\tilde{\mathbf{c}}_1\right\|^2. \tag{19}$$

The subspace that minimizes the MSE has basis vectors which are the eigenvectors of $\mathbf{R}$ associated with the D largest values of $|\mathbf{v}_k^H \mathbf{c}_1 / \lambda_k|^2$, where $\mathbf{v}_k$ is the kth column of $\mathbf{V}$. (Note the inverse weighting of $|\lambda_k|^2$ in contrast with PC.)

This technique, called "cross-spectral (CS)" reduced-rank filtering, was proposed in [42]. This technique can perform well for $D < K$ since it takes into account the energy in the subspace contributed by the desired user. Unlike PC, the projection subspace for CS requires knowledge of the desired user's spreading code $\mathbf{c}_1$. Of course, a disadvantage of eigen-decomposition techniques in general is the complexity associated with estimation of the signal subspace.

Partial Despreading

In this method, proposed in [43], the received DS-CDMA signal is partially despread over consecutive segments of m chips, where m is a parameter. The partially despread vector has dimension $D = \lceil N/m \rceil$ and is the input to the D-tap filter. Consequently, $m = 1$ corre-



▲ *4. Multistage Wiener filter.*

The MMSE DFD has the attractive property that the feedforward filter suppresses other-cell interference, while the feedback filter cancels intra-cell interference.

sponds to the full-rank MMSE filter, and $m = N$ corresponds to the matched filter. The columns of $\mathbf{M}_D$ in this case are nonoverlapping segments of $\mathbf{c}_1$, where each segment is of length m. This allows the selection of performance between that of the matched and full-rank MMSE filters by simply adjusting the number of adaptive filter coefficients.

Multistage Wiener Filter

The multistage Wiener filter (MSWF) was introduced in [44]. Figure 4 shows a block diagram of a four-stage MSWF. The stages are associated with the sequence of nested filters $\mathbf{w}_1, \ldots, \mathbf{w}_D$, where D is the order of the filter. The matrices $\mathbf{B}_1, \ldots, \mathbf{B}_D$ shown in the figure are blocking matrices, i.e.,

$$\mathbf{B}_m^H \mathbf{w}_m = \mathbf{0}. \tag{20}$$

Referring to Fig. 4, let $d_m[i]$ denote the output of the filter $\mathbf{w}_m$ and $\mathbf{r}_m[i]$ denote the output of the blocking matrix $\mathbf{B}_m$. Then the $m+1$st multistage filter is determined by correlating the outputs of the preceding stage

$$\mathbf{w}_{m+1} = E[d_m^* \mathbf{r}_m]. \tag{21}$$

For $m = 0$, we have $d_0[i] = b_1[i]$ (the desired input symbol), $\mathbf{r}_0[i] = \mathbf{r}[i]$, and $\mathbf{w}_1$ is the matched filter $\mathbf{w}_1$. The filter output is obtained by linearly combining the outputs of the filters $\mathbf{w}_1, \ldots, \mathbf{w}_D$ via the weights $a_1, \ldots, a_{D-1}$. The MSWF has the following properties:

▲ At each stage n the filter generates a "desired" sequence $\{d_n[i]\}$ and an "observation" sequence $\{\mathbf{r}_n[i]\}$. At any stage n, if $\mathbf{w}_n$ is replaced by the MMSE filter for estimating $d_{n-1}[i]$ from $\mathbf{r}_{n-1}[i]$, then the resulting filter (with the optimal combining weights) is the full-rank MMSE filter. Each filter $\mathbf{w}_n$ can therefore be viewed as the "matched filter" for the associated estimation problem. The MSWF is constructed iteratively by repeating the same structure, consisting of the matched filter and blocking matrix, at each stage. Continuing this procedure for N iterations gives the full-rank MMSE filter. Terminating after D iterations gives a rank D filter.

▲ Computation of the MMSE filter coefficients does not require an estimate of the signal subspace, as do the eigendecomposition techniques. Successive filters are determined by "residual correlations" of signals in the preceding stage. Adaptive algorithms based on this technique were presented in [45].

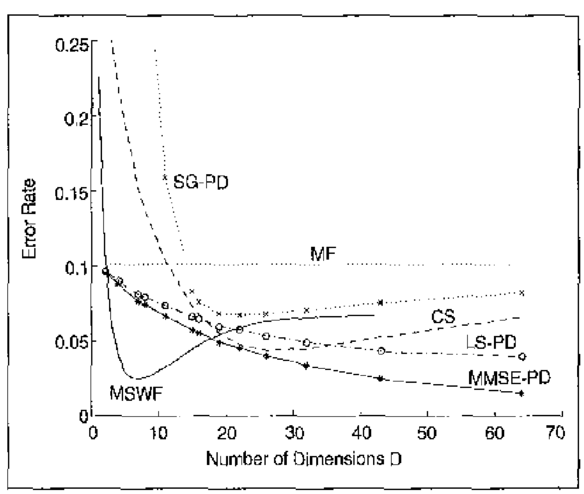
▲ It is shown in [44] that the transformed matrix $\widetilde{\mathbf{R}}$, given by (16) is *tri-diagonal*. That is, it has nonzero elements only along the main diagonal and the adjacent diagonals.

▲ The blocking matrix $\mathbf{B}_m$ is not unique. Although any rank $N - m$ matrix that satisfies (20) achieves the same performance (MMSE), this choice can affect the performance for a specific data record. In particular, a poor choice of blocking matrix can lead to numerical instability.

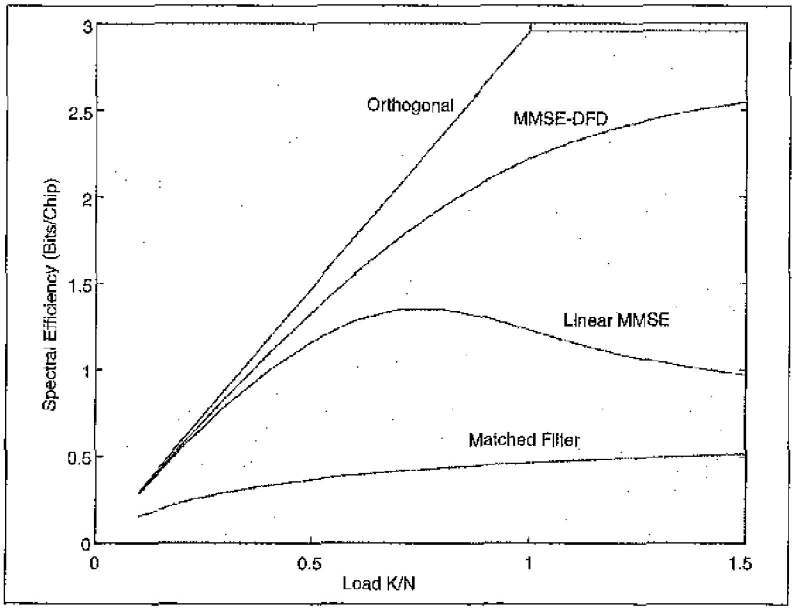
It can be shown that the D-dimensional subspace generated by the rank-D MSWF is the same as the subspace spanned by $\mathbf{c}_1, \mathbf{R}\mathbf{c}_1, \mathbf{R}^2\mathbf{c}_1, \cdots, \mathbf{R}^{D-1}\mathbf{c}_1$ [46]. These vectors are not orthogonal, whereas it can be shown that the basis vectors generated by the MSWF are orthogonal.

A large system analysis of reduced-rank filters, including the MSWF, for synchronous DS-CDMA with randomly assigned spreading codes is given in [46]. Large system analysis of DS-CDMA with random spreading codes was introduced in [47]-[49]. The large system limit is defined by letting the number of users K and processing gain N tend to infinity with fixed load K/N. By using results from the mathematics literature on the distribution of eigenvalues of large random matrices [50], [51], it is possible to compute the large system limit of the output SINR for the full-rank MMSE filter [49]. It has been observed that this limit accurately predicts the performance with moderate K and N (e.g., $N = 32$).

In [46], it is shown that the MSWF has the important property that the rank D needed to achieve a target performance [e.g., output SINR] does not scale with the system size (K and N)]. It is found that $D = 8$ achieves essentially full-rank performance over a wide range of loads K/N. This is in contrast to the other reduced-rank methods discussed, which require that D increase in proportion with K and N to achieve the target performance.



▲ *5. Error rate versus number of dimensions for reduced-rank adaptive algorithms after training with 200 symbols.*



▲ 6. Spectral efficiency versus load for multiuser receivers in synchronous CDMA with $E_b/N_0 = 10$ dB.

Performance Comparison

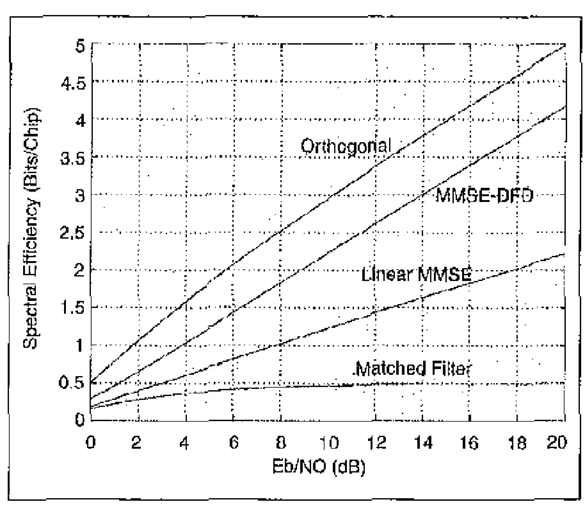
Here we indicate how the different reduced-rank techniques in the preceding section perform when used with a finite-length training sequence. Details on the adaptive algorithms are given in [36] and [45]. Essentially, statistical expectations which occur in the MMSE representations are replaced by sample averages, so that when $D = N$, all algorithms reduce to a full-rank least squares algorithm.

Figure 5 shows error rate versus number of dimensions for reduced-rank adaptive algorithms after training with 200 symbols. In this plot $N = 128$, $K = 42$, and the received powers are log-normal with standard deviation 6 dB, which models received power variations with loose power control. The background SNR is 10 dB. The bit error rate P_e is computed by assuming that the residual interference plus noise at the output of the filter is Gaussian. Results are averaged over random spreading codes, delays, and powers. Curves are shown for the following algorithms: adaptive MSWF, cross-spectral (CS), and the matched filter (MF). In addition three curves are shown for partial despreading with different adaptive estimation methods for the combining coefficients: stochastic gradient (SG-PD), least squares (LS-PD), and MMSE (MMSE-PD). The principal components algorithm performs worse than the cross-spectral method and the corresponding results are omitted from this plot.

Figure 5 shows that the adaptive reduced-rank techniques generally achieve optimum performance when $D < N$. Namely, when D is large, insufficient training data is available to obtain accurate estimates of the filter coefficients, whereas for small D, the filter has insufficient degrees of freedom with which to suppress interference. The minimum error rate for the MSWF algorithm is achieved with only eight stages (dimensions), which is much smaller than the minimizing order for the other reduced-rank techniques. Further- more, this minimum error rate for the MSWF algorithm is substantially lower than the error rate for the matched filter receiver, and is not very far from the full-rank MMSE error rate. Additional simulations with only 100 training samples show that the minimum error rate for the MSWF algorithm is again achieved with $D = 8$, which is consistent with the large system analysis in [46] mentioned earlier.

Adaptive Nonlinear Multiuser Detection

For the reverse link of a cellular network, the objective is to demodulate all users in the corresponding cell in the presence of other-cell interference, noise, and channel and receiver impairments. In this section, we discuss nonlinear decision-feedback receivers for the reverse link which combine multiuser detection of intra-cell users with interference suppression of other-cell users. These techniques are not appropriate for the forward link of a cellular system, since the objective there is to demodulate a *single* user in the presence of interference due to simultaneous transmissions to other users (both inside and outside the cell of interest), noise, and other impairments. Consequently, for the forward link, the *interference suppression* techniques previously discussed are appropriate, rather than multiuser detection.



▲ 7. Spectral efficiency versus E_b/N_0 for multiuser receivers in synchronous CDMA with load $K/N = 1$.

Fundamental Limits

To understand the potential benefits of nonlinear multiuser detection for the reverse link, we first present some fundamental limits on the performance associated with linear and nonlinear receivers. The large-system Shannon capacity for synchronous CDMA with random spreading, additive white Gaussian noise, and optimal (maximum-likelihood) detection was evaluated in [47] and [48]. The *sum* capacity, or capacity summed over all users, is computed, assuming single-user coders and decoders. The "channel" in this analysis consists of the combined synchronous CDMA channel and receiver filter which produces soft outputs. The soft outputs are passed to the single-user decoders. As explained, large-system analysis lets K and N tend to infinity with fixed K/N. Linear receivers are also considered in [48], and the extension to multiuser decision-feedback receivers has been presented in [52]-[54].

Figures 6 and 7 show spectral efficiency versus load K/N and spectral efficiency versus E_b/N_0 for the matched filter, the linear MMSE filter, and the MMSE DFD to be described. In the former plot, $E_b/N_0 = 10$ dB, and in the latter plot, $K/N = 1$. These plots were generated using the results in [48], [53], and [54]. Spectral efficiency refers to the *total* number of bits per chip, summed over all users, which can be reliably transmitted. Also shown is the analogous curve corresponding to orthogonal multiple access. The latter corresponds to the single-user bound since there is no MAI. An important property of the MMSE decision-feedback detector (DFD), to be described, is that it achieves the same sum capacity as the optimal multiuser receiver [52].

These results indicate that:

▲ The linear MMSE detector is nearly optimal for a wide range of loads ($K/N < 70\%$).

▲ At very high loads, and with sufficient E_b/N_0, the MMSE decision-feedback receiver offers a significant performance improvement relative to the linear MMSE receiver.

▲ The capacity of DS-CDMA with the MMSE-DFD (equivalently, maximum-likelihood detection) is close to the capacity of an orthogonal multiple access scheme.

▲ The spectral efficiency of the matched filter reaches an asymptotic limit as $E_b/N_0 \to \infty$, whereas the spectral efficiencies of the linear and MMSE-DFD multiuser detectors increase without bound.

We conclude that for low to moderate loads and power constraints (E_b/N_0), linear MMSE detection can achieve most of the available gain due to multiuser detection. Given sufficient power (E_b/N_0), however, significantly higher spectral efficiencies are achievable with nonlinear techniques. Conversely, at very high spectral efficiencies and loads, nonlinear techniques enable a significant power savings relative to linear techniques.

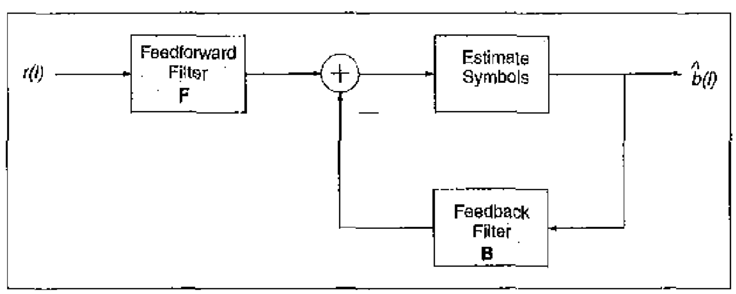
MMSE Multiuser Decision-Feedback Detection

Multiuser decision-feedback for DS-CDMA was first proposed in [55] and [56] and was motivated by earlier work on multichannel (multi-input/multi-output) decision-feedback equalizers [57]-[59]. To simplify the presentation, for now we assume synchronous CDMA with an ideal (AWGN) channel, and defer the extension to asynchronous CDMA with multipath until later in this section. Figure 8 shows a block diagram of a multiuser DFD for synchronous CDMA. The input to the decision device at time i is

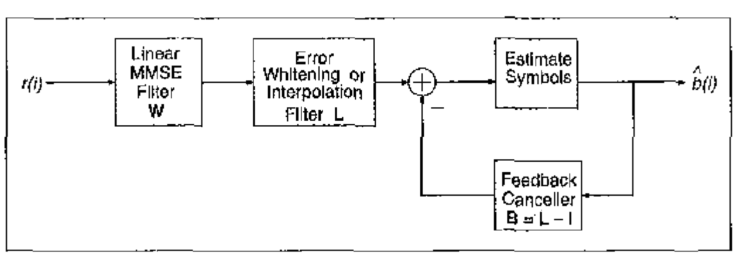
$$\mathbf{y}[i] = \mathbf{F}^H \mathbf{r}[i] - \mathbf{B}^H \hat{\mathbf{b}}[i] \tag{22}$$

where $\mathbf{r}[i]$ is the $N \times 1$ received vector of chip matched-filter outputs corresponding to symbol i, and $\hat{\mathbf{b}}[i]$ is the $K \times 1$ vector of decisions at the output of the decision device. The feedforward matrix $\mathbf{F}$ is $N \times K$, and the feedback matrix $\mathbf{B}$ is $K \times K$ and is typically constrained to have zeros along the diagonal to avoid cancelling the desired symbols.

Determination of the matrices $\mathbf{F}$ and $\mathbf{B}$ depends on the constraints and cost criterion. Namely, a lower diagonal matrix $\mathbf{B}$ corresponds to successive decision-feedback, whereas a full $\mathbf{B}$, except for the diagonal, corresponds to parallel decision-feedback. In the former case (Successive-, or S-DFD), the users are demodulated successively, ideally in order of decreasing power. In this way for each user, only interference from stronger users is cancelled. For the parallel-decision feedback detector (P-DFD), all decisions are fed back simultaneously so that the initial tentative estimates $\hat{\mathbf{b}}$ must be obtained without cancella-



▲ 8. Multiuser decision-feedback detector for synchronous CDMA.



▲ 9. Multiuser decision-feedback detector with error whitening or interpolation filter.

Significant progress has been made during the past decade in overcoming obstacles which have prevented the introduction of multiuser detection in commercial CDMA systems.

tion (e.g., from a linear MMSE receiver). Of course, this procedure can be iterated.

Subject to the preceding constraints on **B**, the matrices **F** and **B** can be selected to minimize the MSE

$$\mathcal{M} = E\{\|\mathbf{b} - \mathbf{y}\|^2\} \tag{23}$$

where $\mathbf{b}[i]$ is the vector of transmitted symbols at time i. Performing this minimization, assuming perfect feedback ($\hat{\mathbf{b}} = \mathbf{b}$), gives the filter structure shown in Fig. 9. Namely, the feedforward filter has the form $\mathbf{F} = \mathbf{W}_{lin}\mathbf{L}$, where $\mathbf{W}_{lin}$ is the linear MMSE filter, and **L** depends on the constraints on **B**. Namely, for the S-DFD, **L** is an error whitening filter (see [60], [61, Sec. 7.5]), whereas for the P-DFD, the columns of **L** can be interpreted as error interpolation filters [62].

With perfect feedback it is easily shown that the MMSE feedback filter perfectly cancels interference from the associated users. Consequently, the feedforward filter for user k turns out to be the linear MMSE filter for that user with the "cancelled" users removed. This implies that for an isolated cell, **F** for the MMSE P-DFD consists of a bank of matched filters, and **B** is the cross-correlation matrix. In other words, the MMSE P-DFD for an isolated cell reduces to the conventional interference canceller proposed and analyzed in [63], [60], and [64]. We also remark that other cost criteria give a structure analogous to that shown in Fig. 9. For example, the feedforward filter for the zero-forcing, or decorrelating DFD consists of the zero-forcing linear filter (decorrelator) followed by an error whitening or error interpolation filter. Finally, other related DFD structures have been considered in [65]-[67].

To summarize, the MMSE DFD has the attractive property that the feedforward filter suppresses other-cell interference, while the feedback filter cancels intra-cell interference. Furthermore, it does not require remodulation for interference cancellation. The main drawback of the DFD is error propagation, which can significantly compromise performance at high error rates.

Adaptive Decision-Feedback Detection

In analogy with linear interference suppression, with short (repeated) spreading codes, the MMSE filters **F** and **B** can be estimated given only a training sequence. Knowledge of spreading codes is not required. Adaptive least squares and stochastic gradient algorithms for accomplishing this have been presented in [68]-[70]. Examples of convergence curves, taken from [70], are shown in Fig. 10. The filters are trained with the number of bits shown. The bit error rate (BER) is then measured over an additional 150 bits, and the results are averaged over many runs. Although the DFDs require more samples to train than the linear receiver, the asymptotic bit error rate is lower. These results take into account error propagation and assume three-path Rayleigh fading. In the absence of error propagation, the asymptotic performance for the P-DFD is the single-user bound.

If the receiver has knowledge of spreading codes of the intra-cell users, as would be expected at the base station, then the DFD filters can be computed directly using this information. This approach is combined with channel estimation in [71] to estimate S-DFD filters in the presence of multipath. An advantage of this approach, relative to the training-based approach, is that fewer data may be needed to obtain accurate estimates of the DFD filters. A disadvantage is that other-cell interference is treated as background noise and is therefore not suppressed. In general, we observe that any of the adaptive techniques discussed earlier for the linear receiver can also be applied to a DFD, where any necessary side information must be provided for *all* users to be demodulated.

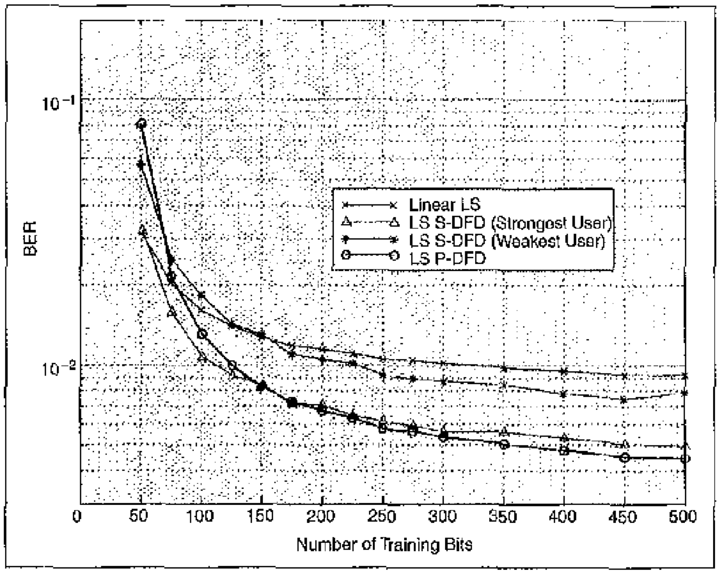
Asynchronous CDMA with Multipath

A requirement of the DFD is that the observation window and sampling times must be the same for all demodulated users. This is in contrast with linear receivers in which the timing can be adjusted separately for each user. Still, asynchronous CDMA can be accommodated by expanding the window of observation to include multiple received symbols. Of course, the received vector corresponding to each desired user must be contained within this interval. If the received signals are chip-asynchronous, then the different timing offsets across users becomes an issue. However, in principle, this can be solved with fractional chip sampling combined with some excess bandwidth (i.e., see [72] and [73]).

In analogy with the linear MMSE receiver, the MMSE DFD filters for asynchronous CDMA are IIR, even in the absence of multipath. Consequently, we must represent the filters as $\mathbf{F}(z)$ and $\mathbf{B}(z)$, each of which has a matrix impulse response. For the general case with multipath, the MMSE solution for $\mathbf{F}(z)$ and $\mathbf{B}(z)$ can be inferred from the results in [59]. Namely, the matrix transfer functions are determined by a spectral factorization of the equivalent multi-input/multi-output channel response.

For adaptive estimation it is convenient to approximate $\mathbf{F}(z)$ and $\mathbf{B}(z)$ as FIR filters. In the absence of multipath, near MMSE performance is generally attainable if the filters span three symbol intervals. This remains true when multipath is present, provided that the delay spread is small relative to the symbol interval. Of course, if the multipath spans multiple symbols, then more memory is required in the DFD to equalize the multi-channel intersymbol interference. The latter situation may arise for high data rate services, which use a relatively small processing gain.

A simple approach to estimating the FIR DFD filters adaptively is described in [70]. If the filters span three symbol intervals, then the received vector $\tilde{\mathbf{r}}[i]$ is formed by stacking $\mathbf{r}[i-1], \mathbf{r}[i]$, and $\mathbf{r}[i+1]$. The sequence of stacked vectors $\{\tilde{\mathbf{r}}[i]\}$ is then the input to an embedded DFD, which has the $3N \times K$ feedforward matrix $\widetilde{\mathbf{F}}$. The output of $\widetilde{\mathbf{F}}$ is stacked in a similar way, and is used to compute the $3K \times K$ feedback matrix $\widetilde{\mathbf{B}}$. Of course, increasing the dimension of the filters in this way means that many more coefficients must be estimated.



▲ 10. Bit error rate versus number of training iterations for adaptive multiuser DFDs. The processing gain $N = 8$, there are four asynchronous users, and $E_b/N_0 = 6$ dB. The channel for each user consists of three Rayleigh fading paths (power profile [0 dB, -2 dB, -4 dB]).

Coded Performance

The results in Figs. 6 and 7 apply to a coded system and assumes an S-DFD with successive decoding [52]. There is relatively little work so far on the performance of MMSE DFDs with actual codes, although some results are presented in [62], [70]. Those results indicate that the S-DFD can offer a substantial gain in performance relative to a linear receiver provided that the users vary substantially in power. For example, in third-generation systems, received power is proportional to rate. This causes a significant power imbalance which an S-DFD can exploit.

Conclusions

Significant progress has been made during the past decade in overcoming obstacles which have prevented so far the introduction of multiuser detection in commercial CDMA systems. The use of short codes, in particular, enables adaptive solutions that require little side information. We have indicated how these solutions can be extended to exploit multipath diversity even without training or explicit channel estimates. Convergence and tracking still remain issues in the presence of bursty interference and moderate to fast fade rates. Reduced-rank methods may be useful in these situations, and this observation is stimulating work along these lines. Finally, we have discussed multiuser decision-feedback techniques, which can also be made adaptive with short codes and can offer significant benefits relative to linear receivers.

Of course, significant challenges still remain, and multiuser detection continues to be an active area of research. The topics discussed in this article represent a subset of topics within multiuser detection which are currently being studied by many investigators. In particular, we have not discussed many practical issues with receiver design and MD. Additional issues, such as the role MD can play in the support of integrated services with different information rate and quality of service requirements, are just emerging.

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