Spot Markets for Spectrum Measurements

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***Abstract*—The recent framework for spectrum sharing in the 3.5 GHz band allows for Environment Sensing Capability operators (ESCs) to measure spectrum occupancy so as to enable commercial use of this spectrum when federal incumbent users are not present. Each ESC will contract with one or more Spectrum Access Systems (SASs) to provide spectrum occupancy data. Commercial firms using the band will in turn contract with a SAS to determine when it can access the spectrum. Initially, the decisions of which ESC and SAS to partner with will likely be based on long-term contracts. In this paper, we consider an alternative framework, in which an ESC sells its spectrum management information via a spot market so that from period-to-period a commercial user can select a different ESC from which to acquire spectrum measurements. We develop a game theoretic model to analyze such a market and show that using such a spot market may better enable multiple commercial firms to operate in a given spectrum band. We also show that this increased competition may not benefit consumer surplus unless firms adopt a non-stationary strategy profile.**

I. Introduction

Recently, the U.S. FCC has finalized plans for the Citizens Broadband Radio Service (CBRS) [1]. These plans enable commercial users to use the 3.5GHz band (3550-3700 MHz) when incumbent users (e.g., federal users and fixed satellite users) are absent. Accessing this band in a given location is to be controlled by one or more *Spectrum Access Systems* (SASs), which are geographic databases that contain information about the locations and spectrum utilization of users of this spectrum. It is envisioned that in many areas, multiple companies will operate approved SASs.[1] Companies wishing to offer service in that band must then register with one SAS. Additionally, each SAS can utilize an *Environmental Sensing Capability operator* (ESC). An ESC will deploy a network of sensors to detect the presence (or absence) of federal incumbent users, enabling firms to better utilize the spectrum than would be possible under more conservative exclusion zones. Again multiple ESCs may operate in a given location.[2]

An interesting feature of the CBRS ecosystem is that there are multiple levels of competition that may emerge. At one level, one or more ESCs may compete to sell their spectrum measurement data to different SASs. Different SASs may in turn compete to sell their service to different firms seeking to utilize the spectrum. Finally, these end firms may compete with each other to offer wireless services to end users. Here, following [4]–[6], we consider a reduced model of such an ecosystem in which ESCs seek to sell spectrum measurements to Spectrum Access firms (SAs) who in turn serve end users. In particular, we do not explicitly model the SAS tier. One can view this as a model in which either the ESC and SAS act as a single company or one in which a SA represents a SAS operator that directly serves many small customers. For such a reduced model, [4]–[6] studied the resulting competition in a static model, i.e., in a model where each SA needed to select a single ESC to use over the time period of interest. In this paper, we consider an alternative market structure in which ESCs sell their measurements via a spot market, so that an SA may dynamically change which ESC it purchases spectrum measurements from over time. Our goals are to understand the potential economic impacts of such a market structure. Would this improve consumer welfare? Would it result in more or less competition? Would promoting such a market be a useful policy to adopt?

We adopt a model similar to that in [4]–[6] in which two SAs in a given area compete to serve a common pool of end users using a shared band of spectrum (e.g. this could model the Generalized Authorized Access (GAA) tier in CBRS). At each time-period, to serve customers an SA must first purchase spectrum measurement data from an ESC. In [4]–[6], this ESC selection was a one-time static decision. Here, instead of a long term contract, we consider a scenerio in which the ESCs sell their information in a spot market. To model a spot market, we first consider the case in which each SA adopts a stationary randomized strategy that specifies the probability of it acquiring information from any of the ESCs in the market at each time period. We also allow for the possibility that a SA may opt to not obtain any information with a non-zero probability at a given instance of the market. If a SA does not obtain information from any of the ESCs, we assume that it can not offer service to the customers. Compared to the static case, in [4]–[6], allowing for such randomized strategies significantly complicates the analysis as the resulting profit functions are non-convex in the ESC selection probabilities. We subsequently also consider non-stationary strategies by viewing the SAs as playing a repeated game.

Given the SA's ESC selection strategy, the two SAs in turn compete to serve end-users. To model the competition among the SAs we adopt a similar framework as that used in [7]–[14] to study competition among wireless service providers using

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[1] In the first round of applications, six different companies requested FCC approval as SAS operators [2].

[2] In the first round, three different ESCs were approved by the FCC [3].

unlicensed and/or licensed spectrum. This in turn is based on models used to study price competition with congestible resources (e.g. [15], [16]). As in these models, we assume that there is a continuum of non-atomic users who in turn select a SA based on the expected payoff of the service. The payoff of service in a given time-slot is given by the difference between the value obtained from service and the sum of the announced service price and a congestion cost that increases in the number of users using the shared band of spectrum (modeling the quality of service received by the users). During times when a SA is unable to serve customers (due to not obtaining information or due to the spectrum not being available) the value its customers receive is zero. The expectation is then taken over time and depends on the SA's ESC selection strategy and the information obtained from the ESCs. Users then seek to subscribe to the SA offering them the largest expected payoff. Here, we assume that the pricing and user subscription decisions are done on a slow time-scale and so model these as static one-shot decisions. One motivation for this is that due to the larger size of the market for end users, having a spot market for wireless service may be more challenging. Also, this is in-line with current practice in which users make long-term decisions on wireless service providers.[3]

We analyze the resulting multi-stage game and seek to characterize its sub-game perfect equilibria, Assuming that the SAs adopt stationary ESC selection policies, we show that the resulting (second stage) price competition has a unique equilibrium that we characterize (Theorem 1). Using this, we then turn to study the first stage ESC selection policy. Our analysis shows that there is *no* equilibrium where both the SAs obtain information from the same ESC with probability 1 (Theorem 2). When there is only a single ESC in the market, we show that the only equilibrium that is possible is where one of the SAs obtains information from the ESC with probability 1, and the other SA (say, k) obtains information with some probability less than 1 (Theorem 3)). The probability that SA k obtains information decreases as the users' valuations increase and may be zero in some cases. Note that for the static market studied in [4], [5], it was shown that multiple SAs will not ever enter a market if there is a single ESC. In contrast, in our spot-market setting both the SAs can co-exist, though, one of the SAs may opt to not obtain information with a non-zero probability. In other words, *a spot market can help to encourage competition in the SA tier of the market.* However, our analysis also reveals that the end user's do not benefit from this increased competition. Even though multiple SAs may exist in the market, the user surplus is always zero. If we allow for non-stationary strategies this may change. Namely, we shown that in an infinitely repeated game, there exists an equilibrium strategy profile where the user's surplus is positive. In that equilibrium strategy profile, the two SAs take turns in purchasing information from the ESC so that in a given time-slot only one purchases information.

We also consider a setting in which there are two ESCs in the market, which may offer different quality of spectrum measurements at different costs. In such a setting, we show that despite this increased competition, the users' surplus remain zero (Theorem 6). This can be contrasted with the one-time static market in [4], [5], where with two ESCs, user surplus can be positive. Hence, from a user surplus perspective, a longer term market may be preferred to a spot-market. Another distinction with the static one-time market in [4], [5] involves the equilibrium ESC selection. In the static case, the only equilibria in which both of the SAs enter the market are those in which each obtains information from a different ESC. For a spot-market, we show that multiple SAs may obtain information from the same ESC if a certain normalized cost of obtaining information from that ESC is not higher compared to obtaining information from the other ESC (Theorem 5). Thus, a spot market may not support competition among ESCs as well as a longer term static market.

Our work enriches the growing literature that studies the role of information acquisition on competition. For example, [17] considers acquiring information about a competitor's supply in a spectrum sharing scenario, and [18] studies firms that can acquire information about customer demand from a third party. The question of whether firms should share information with competitors has also received attention (e.g [19]). Another line of related work addresses issues around the design of the ESC and SAS infrastructure. This includes work on sensor deployment (e.g. [20]), channel assignments (e.g. [21], [22]) and work on privacy issues raised by such networks (e.g. [23], [24]).

II. System Model

We consider a model in which there are *at most* two ESCs (denoted by ESC A and ESC B) and two SAs denoted by SA 1 and SA 2.[4] Each SA seeks to serve users in a given band of spectrum at a given location. To do this, the SAs must acquire spectrum measurements from one of the ESCs and can only use the spectrum when the ESC indicates that it is available (i.e., not being used by a federal incumbent). If an SA does not acquire information from either ESC, we assume it can not serve any users.[5] If both SAs receive information that the spectrum is available, then they both can utilize it. We next discuss the participants in this market in more detail.

A. Information Selling from the ESC

Each ESC provides a binary indication of whether the spectrum is available for use over time based on their own sensing capabilities. We assume that each ESC must be certified to have a negligible probability of missed detection of the incumbent, i.e., if the incumbent is present, the ESC will never announce that the spectrum is available. However, we do allow the ESCs to incur false alarms, i.e., if the incumbent

[3]Considering a spot market for wireless service is also of interest, but left for future work.

[4]Our model can be easily generalized to the scenario where there are more than 2 ESCs; generalizing to more than two SAs is less straightforward and left for future work.

[5]This could model a setting in which the SAs are located in an area near the coasts for CBRS and so excluded from using the spectrum without ESC input.

is not present, an ESC may still announce that the spectrum is not available. An ESC with better sensing capabilities will be less likely to make such errors. We identify each ESC A and B with a probability q_A and q_B, respectively, that gives the probability that the ESC indicates that the spectrum is available (which in turn depends on the incumbent's usage patterns and the ESC's sensing capability). Without loss of generality, we assume that $q_A \geq q_B$ so that ESC A has the higher quality of information (unless they are identical). Further to simplify our exposition, we assume that ESC B's measurement is a degraded version of ESC A, so that whenever the ESC B indicates the channel is available, ESC A also does the same. However, when ESC A estimates the channel is available, ESC B may not estimate the same.[6]

We assume a spot market where the market operates at different time slots $t = 1, \ldots, T$. At each time slot, ESC A (B) sells its prediction to any of the SAs at the price $\tilde{p}_A$ ($\tilde{p}_B$). The SAs do not learn the outcome of the prediction until they make a purchase. Here, $q_A, q_B, \tilde{p}_A$, and $\tilde{p}_B$ are common knowledge to both the ESCs and to the SAs. Throughout this paper, we assume that $\tilde{p}_A$ and $\tilde{p}_B$ are exogenous parameters and focus on the strategic decisions of the SAs given these prices.

B. SAs Decisions

Each SA must make two decisions. First, it must decide whether to acquire information from ESC A, ESC B, or to not acquire any information at all in each time slot.[7] In the first stage, both the SAs simultaneously decide on acquiring information. We initially consider the case where the SAs employ a *stationary memoryless randomized strategy*, i.e., each SA employs the same randomized strategy at every instance irrespective of the decision made by the SAs in the previous slots. Note that though the strategy is static, it must be optimal against the strategy employed by the other SA. SA may select one of the ESCs w.p. 1 (i.e., deterministically obtains information from one of the ESCs). Mathematically, SA i selects ESC j with probability $\sigma_{i,j}$ at each time slot. We allow $\sum_j \sigma_{i,j}$ to be less than 1, which means there is a non-zero probability that the SA i does not choose any of the ESCs. Note that an equilibrium in the stationary distribution is also an equilibrium when the decisions may depend on the history and are the only possible equilibrium when the game is repeated *finitely many times*. We also provide an equilibrium where the decision of the SAs may depend on the history of decisions in an in infinitely repeated game in Section V-A1 under the assumption that there is only one ESC.

The second decision an SA must make is the price p_i that it will charge users for its service. As is the case in the wireless market today, we view the price p_i as representing the amount users pay for receiving long-term service from SA i (e.g., the monthly service price). As such these prices represent the service from an SA averaged over this service period. Here we view these as flat-rate prices, and assume that each SA only offers a single service plan (which is reasonable as in our model the user population is homogeneous).

Each SA i, seeks to maximize its expected profit which is given by

$$\Pi_i = p_i \lambda_i - \sum_j \sigma_{i,j} \tilde{p}_j \tag{1}$$

where λ_i indicates the number of users SA i serves. Here, $\sum_j \sigma_{i,j} \tilde{p}_j$ is the total expected price the SA pays to the ESCs for acquiring information. If SA i decides not to acquire information in the first stage, then we set $\tilde{p}_j = 0$ and $\lambda_i = 0$ so that the overall profit is also zero, i.e., this models a case where SA i decides not to enter the market. This may occur when the revenue the SA would generate is not sufficient to recover the cost of acquiring information from one of the ESCs.

C. User's Subscription Model

We consider a mass Λ of non-atomic users, so that we have $\lambda_1 + \lambda_2 \leq \Lambda$. The users are homogeneous in that each user obtains a value v for getting service from either SA in a given time period. However, users also incur a cost for using the service, which as in [8]–[11] is given by the sum of the price charged to them by the SA and a *congestion cost* they incur when using this service. The congestion cost models the degradation in service due to congestion of network resources. We model the congestion cost for using a band of spectrum with bandwidth W by $g(x/W)$, where x is the total mass of users using that band and g is a convex, increasing function. Hence, the instantaneous pay-off of a user receiving service from SA i is given by

$$v - p_i - g(x/W). \tag{2}$$

The dependence of g on W models the fact that a larger band of spectrum is able to support more users. The mass of users, x, using the band depends in turn on the licensing policy and the information available to the SAs. Here, we focus on shared access, in which case users of both SAs utilize the same band of spectrum whenever both of them know that the spectrum is available. We model this as in [8]–[10], by setting $x = \lambda_1 + \lambda_2$. If there were more than 2 SAs using this band at a given time, then this can be extended naturally by setting $x = \sum_{\imath \in A} \lambda_i$, where A indicates the set of SAs using this band.

The SAs knowledge of spectrum availability in turn depends on the information they acquire from the ESCs. In particular, if SA i obtains information from ESC k and has λ_i users, these users are only able to use the spectrum when the ESC k reports the spectrum is available (which occurs with probability q_k). When users can not use the spectrum, we assume their instantaneous pay-off is zero. When users can use the spectrum, they receive a pay-off as in (2), where the traffic of the other SA will in turn depend on the information that SA receives from its ESC. Hence, the pay-off obtained will be a random variable. We assume that users seek to

[6]Our analysis can easily be extended to the case where instead ESCs A and B make independent errors.

[7]We do not consider SAs purchasing information from both ESCs in a given time-slot. Given our assumption about ESC B's measurement being a degraded version of ESC A's, there is no advantage to doing this. If these measurements experienced independent errors, there could be gain from doing this, which we leave for future work.

maximize the expected value of this quantity.[8] Furthermore, users can choose not to purchase service from either SA, giving them a pay-off of zero.

The specific form of the average congestion will depend on which ESCs the SAs contract with. If both SAs obtain information from the same ESC j, then both SAs' customers will use the spectrum during the times ESC j specifies it is available (which occurs with probability q_j). Note that the probability thatboth the SAs obtain the information from the ESC j is $\prod_i \sigma_{i,j}$. Thus, the expected payoff of users of SA i given that both the SAs obtain information from the same ESC j is given by

$$q_j v - q_j g\left(\frac{\lambda_1+\lambda_2}{W}\right) - p_i \tag{3}$$

where the above occurs with probability $\sigma_{1,j}\sigma_{2,j}$.

Next suppose that the SAs obtain information from different ESCs. First, assume that SA i obtains information from ESC A and SA k obtains information from ESC B. Recall that when ESC B estimates that the channel is available, then ESC A also estimates that the channel is available. Thus, the subscribers of SA k always face congestion from SA i's users. However, the subscribers of SA i only face congestion from SA k's customers when ESC B also indicates that the incumbent is not present (which occurs with probability q_B). Hence, the subscribers of SA i enjoy an exclusive access to the spectrum with probability $q_A - q_B$. This results in the users of SA i having an expected pay-off of

$$q_A v - (q_A - q_B) g\left(\frac{\lambda_1}{W}\right) - q_B g\left(\frac{\lambda_1+\lambda_2}{W}\right) - p_i \tag{4}$$

and the above occurs with probability $\sigma_{i,A}\sigma_{k,B}$.

Similarly, if SA k obtains information from ESC A and SA i obtains information from ESC B, the expected payoff of users of SA i is

$$q_B v - q_B g\left(\frac{\lambda_1+\lambda_2}{W}\right) - p_i \tag{5}$$

and this occurs with probability $\sigma_{i,B}\sigma_{k,A}$.

Finally, suppose one SA i obtains information from ESC j, while the other SA k chooses not to acquire information from either ESC (and so does not serve any customers). The users of SA i then obtain an expected pay-off of

$$q_j v - q_j g\left(\frac{\lambda_i}{W}\right) - p_i \tag{6}$$

which occurs with probability $\sigma_{i,j}(1-\sum_j \sigma_{k,j})$. Note that $\sum_j \sigma_{k,j}$ denotes the probability that the SA k does not obtain information from any of the ESCs. Here, we assume that the users have to pay the price p_i if they subscribe to the SA i even when SA i does not obtain any information from any of the ESCs.

[8]For example, this is reasonable when users are purchasing service contracts with a long enough duration so that they see many realizations of the ESC reports.

Thus, the expected-payoff of users of SA i is

$$\begin{aligned}\Pi_i = &\sum_j \sigma_{1,j}\sigma_{2,j}\left(q_j v - q_j g\left(\frac{\lambda_1+\lambda_2}{W}\right)\right)\\ &+ \sigma_{i,A}\sigma_{k,B}\left(q_A v - (q_A-q_B) g\left(\frac{\lambda_i}{W}\right) - q_B g\left(\frac{\lambda_1+\lambda_2}{W}\right)\right)\\ &+ \sigma_{i,B}\sigma_{k,A}\left(q_B v - q_B g\left(\frac{\lambda_1+\lambda_2}{W}\right)\right)\\ &+ \sum_j \sigma_{i,j}(1-\sigma_{k,A}-\sigma_{k,B})\left(q_j v - q_j g\left(\frac{\lambda_i}{W}\right)\right) - p_i.\end{aligned} \tag{7}$$

Here, we use k to denote the other SA from i.

D. Multi-Stage Market Equilibrium

We model the overall setting as a game with the SAs and the users as the players. Each SA's pay-off in this game is its profit (cf. (1)), while each user's objective is the expected pay-off described in Section II-C. This game consists of the following stages:

1) In the first stage, each SA i selects $\sigma_{i,j}$ for each ESC j. Note that $\sigma_{i,j}$ can be 0 for all j in the scenario where SA i stays out of the market.
2) In the second stage, SA i selects its price p_i knowing the decisions made in stage 1 by both SAs.
3) In the last stage, given the first two stages' decisions, the subscribers will choose one of the SAs from which to receive service or choose not to receive service.

We refer to a sub-game perfect Nash equilibrium of this game as a *market equilibrium*. We again emphasize that we can view the first stage's randomized strategy as being played over multiple time-slots to yield the corresponding average pay-off, while the second and third stage decisions are performed once in the time-scale of interest.

III. Third Stage Equilibrium

In the final stage of the game, the *user equilibrium* specifies the subscribers λ_i of each SA i given the prices selected in the second stage and the ESC choices made in the first stage.

Each user is seeking to maximize its expected pay-off. Given our assumption of identical non-atomic users, the user equilibrium can be characterized as a *Wardrop equilibrium* [25].

Definition 1. ***Wardrop Equilibrium****: In a Wardrop equilibrium, only SAs whose expected payoff is maximum (among all SAs) and non-negative will serve any positive mass of users.*

In other words, if in equilibrium both SAs are serving customers, then the expected pay-offs for both SA's must be the same (since, otherwise some customers would switch to the other SA). If one SA is not serving any customers, then its expected pay-off must be smaller than that of the other SA. Additionally, this expected pay-off must be non-negative as otherwise some customers would be better off not purchasing service.

Note that if fewer than Λ customers are receiving service, then it must be that the expected pay-off is equal to zero as otherwise, some customers not receiving service would choose to receive service since the congestion cost is increasing in the number of users. Hence, we obtain the following–

Lemma 1. *The user's surplus (or, user's expected payoff) is positive only if* $\lambda_1 + \lambda_2 = \Lambda$.

Note that even though $\lambda_1 + \lambda_2 = \Lambda$ it may happen that the expected payoff is zero. In the following we focus on the case where both SAs always serve the entire population of Λ customers, i.e..

Assumption 1. *We assume that under any best response price from one of the SAs (provided at least one SA enters the market in stage 1), we have* $\lambda_1 + \lambda_2 = \Lambda$.

Specifically, we characterize the equilibrium where all the users are served. From the preceding discussion such equilibria are sufficient to characterize if the user surplus can ever be positive.

Note that if the users subscribe to both the SAs, the expected payoffs from the two SAs must be equal. Hence, we have $\lambda_1 > 0$ and $\lambda_2 > 0$ only if

$$\Pi_1 = \Pi_2. \tag{8}$$

Next we give a condition under which one of the SAs does not serve any customers.

Lemma 2. *If both SAs obtain information with probability one from the same ESC (i.e.,* $\sigma_{1,A} = \sigma_{2,A} = 1$ *or* $\sigma_{1,B} = \sigma_{2,B} = 1$*), then* $\lambda_i = 0$ *if* $p_i < p_j$ *where* $i \neq j, i, j \in \{1, 2\}$.

This lemma implies that if the SAs obtain information from the same ESC with probability one, then the SA which offers the lowest price will be the only one serving customers. This follows from noting that under these conditions, only one term in (7) will be non-zero each SA and this term will be identical for the two SAs. Hence, the SA with the lowest price, will have the largest expected payoff.

To simplify our analysis in the remainder of the paper we make the following assumption:

Assumption 2. *The congestion cost is linear, i.e.,* $g(x) = \frac{x}{W}$.

The linearity assumption is quite common in the literature [4], [5], [10] and indicates that the congestion is proportional to the load per unit bandwidth. With this assumption, a necessary condition for Assumption 1 to hold is given by the following:

Lemma 3. *Assumption 1 holds only if* $v > \Lambda/W$.

For example if $v = 1$, then this condition becomes $\Lambda/W < 1$. In general this is showing that only if v is large enough compared to the congestion cost, the SAs will serve the entire customer base.

IV. Second Stage Price Equilibrium

We now turn to the second stage in which given their first stage ESC selections ($\sigma_{i,j}$ for all i and j), the SAs set prices to compete for customers. We, first show that in this stage there is a unique price equilibrium (Theorem 1). Subsequently, we show that if the SAs employ the same ESC selection strategy, the market is shared between the two SAs (Corollary 2). We also show that if both the SAs obtain information from the same ESC with probability 1, the prices become zero (Corollary 3). Finally, we characterize the second-stage price strategy when there is a single ESC.

The second stage price equilibrium is achieved by first finding the best response p_i for a given p_j, $j \neq i$ and then solving for the fixed point (p_1^*, p_2^*) of these best responses. The result of this is characterized in the following theorem.

Theorem 1. *If for a given ESC selection strategy, there is a second stage equilibrium in which both SAs are serving customers, then this is the unique second stage price equilibrium and it satisfies:*

$$\begin{aligned}
p_1^* =& \frac{(\sigma_{1,A} - \sigma_{2,A})q_A v}{3} + \frac{(\sigma_{1,B} - \sigma_{2,B})q_B v}{3} + \\
& \frac{q_A(2\sigma_{2,A}(1-\sigma_{1,A}) + \sigma_{1,A}(1-\sigma_{2,A}))\Lambda}{3W} + \\
& \frac{q_B(2\sigma_{2,B}(1-\sigma_{1,A}-\sigma_{1,B}) - 2\sigma_{2,A}\sigma_{1,B})\Lambda}{3W} + \\
& \frac{q_B(\sigma_{1,B}(1-\sigma_{2,A}-\sigma_{2,B}) - \sigma_{1,A}\sigma_{2,B})\Lambda}{3W}
\end{aligned}$$

$$\begin{aligned}
p_2^* =& \frac{(\sigma_{2,A} - \sigma_{1,A})q_A v}{3} + \frac{(\sigma_{2,B} - \sigma_{1,B})q_B v}{3} + \\
& \frac{q_A(\sigma_{2,A}(1-\sigma_{1,A}) + 2\sigma_{1,A}(1-\sigma_{2,A}))\Lambda}{3W} + \\
& \frac{q_B(\sigma_{2,B}(1-\sigma_{1,A}-\sigma_{1,B}) - \sigma_{2,A}\sigma_{1,B})\Lambda}{3W} + \\
& \frac{q_B(2\sigma_{1,B}(1-\sigma_{2,A}-\sigma_{2,B}) - 2\sigma_{1,A}\sigma_{2,B})\Lambda}{3W}.
\end{aligned}$$

The corresponding third-stage Wardrop equilibrium is

$$\lambda_1 = \frac{p_1^* W}{D}, \text{ and } \lambda_2 = \frac{p_2^* W}{D} \tag{9}$$

where

$$\begin{aligned}
D =& q_A(\sigma_{2,A}(1-\sigma_{1,A}) + \sigma_{1,A}(1-\sigma_{2,A})) \\
& + q_B\left((\sigma_{2,B} + \sigma_{1,B})(1-\sigma_{1,A}-\sigma_{1,B})\right. \\
& \left. - \sigma_{2,A}\sigma_{1,B} - \sigma_{1,A}\sigma_{2,B}\right).
\end{aligned}$$

Recall, that under Assumption 1 $\lambda_1 + \lambda_2 = \Lambda$. Also note that if $\sigma_{1,A} > \sigma_{1,B}$, and $\sigma_{2,A} > \sigma_{2,B}$, the prices given in this theorem are larger as v increases. Intuitively, if a SA chooses to obtain information more frequently compared to the other, the users will obtain a higher expected payoff. The expected payoff achieved increases with v, hence, the price of the SA also increases.

Note from (9) that the market share of SA i also increases as the price p_i^* increases. However, since $\lambda_1 + \lambda_2 = \Lambda$, if p_i^* increases p_j^* must decrease.

Next we give a condition to characterize when this theorem applies:

Corollary 1. *A necessary and sufficient condition for Theorem 1 to hold is if*

$$\Pi_1 = \Pi_2 \text{ and } \Pi_i \geq 0 \tag{10}$$

where Π_i is given in (7) with p_i replaced by p_i^* from Theorem 1 and λ_i is replaced by the expressions in (9).

The inequality in (10) ensures that the users' surpluses are non-negative. In other words, one can simply evaluate these expressions and if they satisfy this corrolary, then there is a unique equilibrium as given in Theorem 1.

We next consider the symmetric case in which $\sigma_{1,A} = \sigma_{2,A}$ and $\sigma_{2,B} = \sigma_{2,A}$.

Corollary 2. *If $\sigma_{1,A} = \sigma_{2,A}$, and $\sigma_{1,B} = \sigma_{2,B}$, then the price equilibrium in Theorem 1 becomes*

$$p_1^* = p_2^* = \frac{q_A\sigma_{2,A}(1-\sigma_{1,A})\Lambda}{W} + \frac{q_B(\sigma_{2,B}(1-\sigma_{1,A}-\sigma_{1,B}) - \sigma_{2,B}\sigma_{1,A})\Lambda}{W} \quad (11)$$

$$\lambda_1 = \lambda_2 = \Lambda/2 \quad (12)$$

Thus, the prices and the market shares are also the same for SAs if they employ the same strategy for choosing ESCs. The prices become independent of v. Intuitively, since both the SAs employ the same strategy, the expected payoff of users are balanced out, hence, the equilibrium prices become equal.

Note that prices increase with an increase in the variance of the spectrum availability. Intuitively, if there is a certainty (i.e. $\sigma_{i,A}$ or $\sigma_{i,B}$ is higher) in the strategy of both the SAs, they will compete more fiercely since they will have similar information or quality of service. Thus, the price increases with an increase in the variance.

We, now, specify the strategy where both the SAs obtain information from the same ESC with probability 1.

Corollary 3. *If $\sigma_{1,A} = \sigma_{2,A} = 1$ (thus, $\sigma_{1,B} = \sigma_{2,B} = 0$) or $\sigma_{1,B} = \sigma_{2,B} = 1$, then $p_1^* = p_2^* = 0$.*

Thus, if both the SAs always obtain information from the same ESC, the prices of both the SAs are zero. Since the SAs must pay a fee to the ESCs, the profits of the SAs will be negative. The above result follows directly from Lemma 2. When the SAs obtain information from the same ESC, the one which sets a lower price can have all the market share, hence, the SAs engage in a "price war," and compete the prices to zero. Note that the prices paid to the ESCs are paid in the first stage and thus are sunk costs in this stage and so do not influence this price war.

Finally, we specify the strategy where there is only one ESC j, $j \in \{A, B\}$ in the market. We can capture this in our formulation by setting $\sigma_{i,l} = 0$ for ESC $l \neq j$ so that $\sigma_{2,A}\sigma_{1,B} = \sigma_{1,B}\sigma_{2,A} = 0$. In this case, for a SA, selecting ESC I is equivalent to not selecting and ESC in agiven period and so we can view their strategy as simply deciding on the probability that they will select ESC j.

Corollary 4. *If there is only ESC j, the unique second stage pricing strategy when both SAs serve customers is*

$$p_1^* = \frac{(\sigma_{1,j}-\sigma_{2,j})q_jv}{3} + \frac{2\Lambda\sigma_{2,j}(1-\sigma_{1,j})}{3W} + \frac{\Lambda\sigma_{1,j}(1-\sigma_{2,j})}{3W}$$
$$p_2^* = \frac{(\sigma_{2,j}-\sigma_{1,j})q_jv}{3} + \frac{\Lambda\sigma_{2,j}(1-\sigma_{1,j})}{3W} + \frac{2\Lambda\sigma_{2,j}(1-\sigma_{1,j})}{3W}. \quad (13)$$

The corresponding third stage Wardrop equilibrium is

$$\lambda_1 = \frac{p_1^*W}{q_j\sigma_{1,j}(1-\sigma_{2,j}) + q_j\sigma_{2,j}(1-\sigma_{1,j})}$$
$$\lambda_2 = \frac{p_2^*W}{q_j\sigma_{1,j}(1-\sigma_{2,j}) + q_j\sigma_{2,j}(1-\sigma_{1,j})}. \quad (14)$$

Corollary 5. *A necessary and sufficient condition for Corollary 4 is that*

$$\sigma_{1,j}\sigma_{2,j}(q_jv - q_j\Lambda/W) + \sigma_{1,j}(1-\sigma_{2,j})(q_jv - q_j\lambda_1/W) \geq p_1^*. \quad (15)$$

The inequality in (15) must be satisfied in order to ensure that the user's surpluses are non-negative unless users will not subscribe (cf. (7)). Since $\Pi_1 = \Pi_2$, thus, it is sufficient to ensure that $\Pi_1 \geq 0$.

Note that in a static market, $\sigma_{i,j}$ is either 1 or 0. In [4], [5], it is shown that in the static market, both the prices of SAs are zero if there is a single ESC. However, in the spot market, the prices of the SAs may be positive in the scenario $\sigma_{i,j} < 1$. *Thus, a SA can choose not to obtain information from the ESC with a positive probability and this can make the equilibrium prices of both the SAs positive.*

If $\sigma_{i,j} < \sigma_{k,j}$, the price of SA i can become negative if v is large, since the co-efficient of v is negative.

Corollary 6. *If $\sigma_{i,j} < \sigma_{k,j}$, $i, k \in \{1, 2\}, i \neq k$, the price of SA i is zero if*

$$v \geq \frac{\frac{\Lambda\sigma_{i,j}(1-\sigma_{k,j})}{W} + \frac{2\Lambda\sigma_{k,j}(1-\sigma_{i,j})}{W}}{\sigma_{k,j}-\sigma_{i,j}}. \quad (16)$$

Note that if v is large, or W is small, at least one of the SAs price will be zero unless $\sigma_{i,A} = \sigma_{j,A}$. If $\sigma_{k,j} = 1$, the right hand side is upper bounded by $\frac{2\Lambda}{W}$.

V. First Stage Equilibrium

We now characterize the first-stage equilibrium in which the SA determine their ESC selection strategies. First, we specify a characteristic of the equilibrium which rules out that the SAs obtain information from the same ESC with probability 1 (Theorem 2) . In Section V-A we characterize the equilibrium strategies where there is only one ESC. Subsequently, we characterize the equilibrium strategies when there are multiple ESCs in Section V-B.

First, we rule out the possibility that both the SAs will always obtain information from the same ESC w.p. 1.

Theorem 2. *There is no equilibrium where $\sigma_{1,j} = \sigma_{2,j} = 1$ for any positive $\tilde{p}_j > 0$.*

The above result readily follows from Corollary 3 since if $\sigma_{1,j} = \sigma_{2,j} = 1$, $j \in \{A, B\}$ the prices must be zero from Corollary 3. Since the SAs have to pay a positive price to ESC j, both the SAs' profits are negative. Any SA can choose not to obtain any information and can secure at least zero profit. Hence, there is a profitable unilateral deviation.

Recall that the profit of SA i is

$$p_i^* \lambda_i - \sum_j \sigma_{i,j} \tilde{p}_j. \tag{17}$$

When both the SAs are serving customers, p_i^* and λ_i are given in Theorem 1. SA i would want to select $\sigma_{i,j}$ in order to maximize its own profit only. Note from the expressions in Theorem 1 that the expression $p_i^* \lambda_i$ is not concave in $\sigma_{i,j}$. Thus, it is challenging to find the Nash equilibrium.

A. Single ESC

We, first, state the *only* possible class of Nash equilibrium strategy profile where there is a single ESC. We also show that the user's surplus is always zero.

We first state a strategy profile and will show that the strategy profile is a NE.

Definition 2. *The strategy profile* ***SP*** *is defined as one in which SA i $i \in \{1, 2\}$ selects ESC j with probability 1 and SA k , $k \neq i$ selects ESC j with probability ϕ where ϕ is the following*

$$\phi = \max\{\frac{2\Lambda/W - v}{2v - \Lambda/W}, 0\} \tag{18}$$

and does not obtain information from ESC j with probability $1 - \phi$.

In the strategy profile SP one of the SAs selects the ESC j with probability 1, and the other SA k selects the ESC j with probability ϕ. Hence, $\sigma_{k,j} = \phi$. In other words, one of the SAs *randomizes* between obtaining information and not obtaining information while the other always obtains information. Hence, the strategy profile **SP** is asymmetric as different SAs employ different strategies. Also note that the strategy profile is *not unique* since SA i and SA k can interchange the strategy.

We, now state the second stage price strategy and the third-stage Wardrop equilibrium under the strategy profile **SP**. The price strategy under the strategy profile **SP** is the following

$$p_i^* = (1 - \phi) q_j (\frac{v}{3} + \frac{\Lambda}{3W})$$
$$p_k^* = (1 - \phi) q_j (\frac{2\Lambda}{3W} - \frac{v}{3}). \tag{19}$$

The corresponding Wardrop equilibrium is

$$\lambda_i = \frac{p_i^* W}{1 - \phi} \quad \lambda_k = \frac{p_k^* W}{1 - \phi}. \tag{20}$$

Thus, the expected payoff of the SAs are

$$\pi_i = (1 - \phi) W q_j (\frac{v}{3} + \frac{\Lambda}{3W})^2 - \tilde{p}_j$$
$$\pi_k = (1 - \phi) W q_j (\frac{2\Lambda}{3W} - \frac{v}{3})^2 - \tilde{p}_j \phi. \tag{21}$$

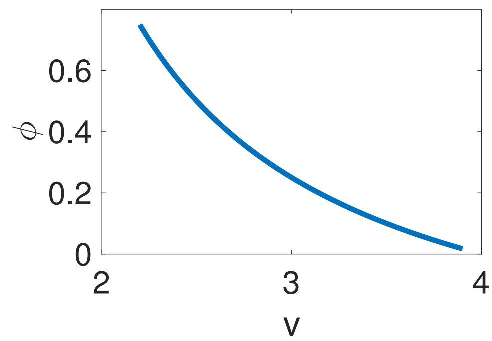

Fig. 1. The variation of ϕ as a function of v in an example scenario with $\Lambda = 300, W = 150, q_j = 0.5$.

Now, we are ready to state the result.

Theorem 3. *1) If $(1 - \phi) W q_j (2\Lambda/(3W) - v/3)^2 \geq \tilde{p}_j \phi$ and $\Lambda/W < v < 2\Lambda/W$ where ϕ is given in (18), the equilibrium strategy is as stated in* ***SP****.*

2) If $(1 - \phi) W q_j (2\Lambda/(3W) - v/3)^2 < \tilde{p}_j \phi$ or $v \geq 2\Lambda/W$, where ϕ is given in (18) then only one of the SAs can exist in the market. SA i, $i \in \{1, 2\}$ will obtain information with probability 1 and the SA $k, k \neq i$ will stay out of the market. The pricing strategy is the same as the monopoly strategy (Theorem 4 of [5]).

There is no other possible equilibrium given Assumption 1.

The strategy profile when only one of the SAs obtains information and the other never does leads to the equivalent static market scenario. In the static market as described in [4], [5], a monopoly always arises when there is a single ESC. Our analysis shows that the monopoly scenario can arise with a spot market if $v \geq 2\Lambda/W$. Hence, if v is large, similar to the static market, a monopolistic outcome will occur when there is a single ESC. However, unlike the static case, when v is not too large, a competitive equilibrium can arise with a spot market.

Note from Lemma 3 that for $v < \Lambda/W$, $\lambda_1 + \lambda_2$ can not be equal to Λ. Hence, the result corresponding to $v < \Lambda/W$ does not appear in the result. Note that ϕ decreases as v increases. Hence, as v increases, one of the SAs will be more likely to opt out against obtaining information from the ESC. Thus, as v increases, a monopoly scenario arises with a higher probability. Figure 1 illustrates this by plotting the variation of ϕ as a function of v for a given scenario.

From (21) that the revenue of SA k which randomizes its ESC selection is given by $(1 - \phi) W q_j (2\Lambda/(3W) - v/3)^2$. If this is less than that SA's expected payment to the ESC, $\tilde{p}_j \phi$, then this SA can not obtain a positive profit. When this occurs then as shown in Theorem 3 that SA will stay out of the market. Hence, for larger values of $\tilde{p}_j$ is is more likely that a monopolistic outcome will occur. Figure 2 illustrates this by plotting the variation of SA k's payoff as a function of $\tilde{p}_j$ for a given scenario.

The expected payoff of SA k is strictly less than that of SA i since $v > \Lambda/W$ (cf.(21)). Note that SA k's profit decreases with an increase in ϕ. However, if ϕ is 0, the profit of SA j will be 0 as SA j can not serve any users. Thus, ϕ must be

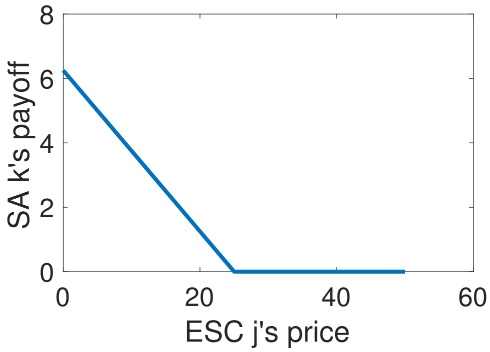

Fig. 2. Variation of SA k's payoff as a function of $\tilde{p}_j$ for the same example as in Figure 1 ($\Lambda = 300, W = 150, v = 3, q_j = 0.5$). When $\tilde{p}_j > 25$, SA k does not obtain information from the ESC.

the minimum possible value such that SA j will exist in the market which is given by the expression in (18). Intuitively, by randomizing SA k differentiates itself from SA i and avoid hence avoid a price war. This shows that a SA will try to differentiate as much as possible while still being profitable.

The next result characterizes the user's surplus.

Corollary 7. *The user's surplus under any market equilibrium is zero.*

Proof. From Lemma 1 note that the users' surplus can be positive only if $\lambda_1 + \lambda_2 = \Lambda$. When $\lambda_1 + \lambda_2 = \Lambda$ the only possible equilibria are given by Theorem 3. From Theorem 4 of [5], under the monopoly strategy, the user's surplus is always zero. Hence, if the case (2) of Theorem 3 is satisfied, the user's surplus is zero. To complete the proof, we consider the first case in which the equilibrium profile is **SP**.

From (7) note that the user's surplus from SA k is

$$\begin{aligned}&\phi(q_j v - q_j\Lambda/W) - p_k\\&= \phi(2q_j v/3 - q_A\Lambda/(3W)) - q_j(2\Lambda/(3W) - v/3)\\&= 0 \quad \text{(from (18))}\end{aligned} \tag{22}$$

Hence, the result follows. $\square$

This result shows that in equilibrium profile **SP**, SA k selects the probability to obtain information from the ESC j in a manner such that the users' surplus becomes 0. Note that when the SAs participate in the long term market, the monopoly scenario arises where the user's surplus is zero and only one of the SAs exists in the market. In the spot market, the user's surplus does not become positive even though both the SAs can participate in the market.

Also note that if W is large, the condition in Case (1) is less likely to be satisfied. Thus, a monopoly scenario more likely occurs when the amount of shared bandwidth is large.

1) Non-Stationary Strategy: We next consider a case in which the SAs are playing an infinitely repeated game and can adopt non-stationary strategies which may depend on the history of play.[9] *We show that under a non-stationary strategy, the user's surplus can be positive.*

[9]We consider an infinitely repeated game, since in the finitely repeated game the stationary strategy profile is the only equilibrium.

We seek to characterize the sub-game perfect Nash equilibrium. The payoff of SA i is the following

$$\sum_{t=1}^{\infty} \gamma^{t-1}\pi_{i,t} \tag{23}$$

where $\pi_{i,t}$ is the profit at stage t, and $\gamma \in (0,1)$ is a discount factor.

We now specify the sub-game perfect equilibrium in the repeated game.

Theorem 4. *Consider the strategy profile* $\textbf{SP}_{\text{multi}}$*: SA 1 (or, SA 2) selects ESC j with probability 1 in odd time slots $t = 1, 3, \ldots$ and selects ESC j with probability 0 in even time slots $t = 2, 4, \ldots$, SA 2 (or, SA 1, resp.) selects ESC j with probability 0 in* odd *time slots $t = 1, 3, \ldots$, and selects ESC j with probability 1 in* even *time slots $t = 2, 4, \ldots$. If SA k $k \in \{1, 2\}$ deviates at time slot t, SA $i \in \{1, 2\}, i \neq k$ will select ESC j with probability 1 and SA k will select ESC j with probability ϕ (cf. 18)) as stated in Theorem 3.*

The above strategy profile constitutes a sub-game perfect equilibrium for high enough $\gamma < 1$.

Thus, in the repeated interaction only one of the SAs will be active at a given time slot. The SAs alternate between serving users and not. If a SA (say, 1) deviates and obtains information in two consecutive time slots, the other SA (say 2) punishes the SA by reverting back to the stationary strategy stated in Theorem 3 where SA 2 will presume the role of i and always obtains information from ESC j with probability 1. SA 1's payoff will be strictly less than what it obtains prior to deviating. Thus, neither SA has any incentive to deviate.

The price of each of the SA i under the strategy profile $\textbf{SP}_{\text{multi}}$ is given by the following

$$p_1^* = p_2^* = q_j\frac{\Lambda}{4W} \tag{24}$$

and under this profile we have $\lambda_1 = \lambda_2 = \Lambda/2$. The users' welfare is $\frac{q_j v}{2} - \frac{q_j\Lambda}{2}$ which is always *positive* if $v > \Lambda$.

If either of the SAs deviates, since the SAs revert back to the strategy profile **SP**, the SPs' equilibrium prices should be given by (19) and λ_i for each SA i is given by (20). However, there is a subtlety here due to the differences in the time-scale over which prices are made and ESC selections are made. When prices are made over a slower time-scale, then an ESC can not instantly change its price when it detects a deviation. Instead, one can view the prices as being repeatedly set in a longer time-scale repeated game, whenever one detects that in the previous round of the pricing game the other SA deviated, then in the next time-period, the prices from profile SP will be adopted.

B. 2 ESCs

We now briefly consider the case when there are 2 ESCs: A, and B. For this section, we again focus on stationary ESC selection policies.

Theorem 5. *If*

$$\frac{\tilde{p}_A}{q_A} < \frac{\tilde{p}_B}{q_B}$$

the equilibrium coincides with the one as stated in Theorem 3 with $j = A$.

Thus, even when there are two ESCs one of the SAs may opt against obtaining any information from any of the ESCs with a positive probability. Specifically, if the cost of obtaining information from ESC B normalized by the channel availability of ESC B is larger than the corresponding normalized cost of ESC A, then no SA will obtain information from ESC B. Note that in the static market considered in [4], [5], when both SAs enter the market, they always obtain information *different* ESCs. This results shows that this need not be true with a spot market. In a static market, the selection of different ESC provides a way for the SAs to differentiate themselves; in aspot market the SAs can instead use the ESC selection policy to accomplish this.

A general characterization of the equilibrium in the two ESC case appears to be cumbersome and is left for future work. However, in general we can show the following:

Theorem 6. *With two ESCs, in any market equilibrium, the user's surplus is always zero.*

In other words, the result that the user's surplus is zero is also true when there are multiple ESCs in the market. Hence, compared to the static market, the spot market cannot generate a positive users' surplus.

VI. Conclusion and Future Work

In this paper we consider a scenario in which spectrum measurements are sold by ESCs via a spot market. We developed a model and characterized the market equilibira for a setting in which two competing SAs first decide on their ESC selection strategy and then compete on price to serve a pool of end users with shared spectrum. We show that unlike the static market considered in prior work, with a spot market multiple SAs can co-exist even when there is a single ESC. However, we show that in the spot market, the user's surplus always remain zero. In the repeated version of the game, there exists a strategy profile where the user's surplus can be positive. This suggests that allowing for such a market may help to increase the number of SAs in the market but may not yield benefits to consumers unless that SAs jointly adopt a non-stationary selection strategy.

Our model can be extended in several directions. We focused on a simple setting with only SAs and at most two ESCs, considering markets with a larger number of SAs and ESCs in one possible future direction. We also only consider the case where the two SAs competed using a single band of shared spectrum; considering other spectrum bands and different licensing policies is another possible future direction. In our model all end users are homogeneous; models with heterogeneous customers would also be of interest.

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APPENDIX

A. Proof Sketch of Theorem 1

In any Wardrop equilibrium where both SAs serve customers, we must have $\Pi_1 = \Pi_2$. Hence, using the expression in (7) it follows that in any such Wardrop equilibrium

$$\begin{aligned} &q_A(\sigma_{1,A}-\sigma_{2,A})v + q_B(\sigma_{1,B}-\sigma_{2,B})v - p_1 + p_2 = \\ &\frac{q_A\lambda_1\sigma_{1,A}(1-\sigma_{2,A}) - \lambda_2\sigma_{2,A}(1-\sigma_{1,A})}{W} + \\ &\frac{q_B\lambda_1(\sigma_{1,B}(1-\sigma_{2,A}-\sigma_{2,B}) - \sigma_{1,A}\sigma_{2,B})}{W} - \\ &\frac{q_B\lambda_2(\sigma_{2,B}(1-\sigma_{1,A}-\sigma_{1,B}) - \sigma_{2,A}\sigma_{1,B})}{W}. \end{aligned} \tag{25}$$

Under Assumption 1, $\lambda_1 + \lambda_2 = \Lambda$. Thus,

$$\begin{aligned} &q_A(\sigma_{1,A}-\sigma_{2,A})v + q_B(\sigma_{1,B}-\sigma_{2,B})v - p_1 + p_2 = \\ &\frac{q_A\lambda_1\sigma_{1,A}(1-\sigma_{2,A}) + (\lambda_1-\Lambda)\sigma_{2,A}(1-\sigma_{1,A})}{W} + \\ &\frac{q_B\lambda_1(\sigma_{1,B}(1-\sigma_{2,A}-\sigma_{2,B}) - \sigma_{1,A}\sigma_{2,B})}{W} - \\ &\frac{q_B(\Lambda-\lambda_1)(\sigma_{2,B}(1-\sigma_{1,A}-\sigma_{1,B}) - \sigma_{2,A}\sigma_{1,B})}{W}. \end{aligned} \tag{26}$$

Using this we can solve for λ_i in terms of p_i and see that SA i's revenue, $p_i\lambda_i$ is a concave function of p_i. The price which maximizes $p_i\lambda_i$ can thus be obtained from the first order optimality conditions. This gives the best response function for SA 1 as

$$\begin{aligned} p_1^* = &\frac{q_A(\sigma_{1,A}-\sigma_{2,A})v + q_B(\sigma_{1,B}-\sigma_{2,B})v}{2} \\ &+ \frac{q_A\Lambda\sigma_{2,A}(1-\sigma_{1,A})}{2W} + \frac{q_B\Lambda(\sigma_{2,B}(1-\sigma_{1,A}-\sigma_{1,B}) - \sigma_{2,A}\sigma_{1,B})}{2W} \\ &+ p_2^*/2. \end{aligned} \tag{27}$$

Due to the symmetry, p_2^* can be obtained by replacing 1 with 2. Solving for the intersection of these best response functions, the result follows.[10]

B. Proof Sketch of Theorem 3

The proof depends on the following Lemma.

Lemma 4. *$p_i^*\lambda_i$ is strictly increasing function in $\sigma_{i,j}$ if $\sigma_{k,j} < 1$ where $i, k \in \{1, 2\}$ and $i \neq k$.*

The above lemma can be proved by differentiating $p_i^*\lambda_i$ (in Corollary 4) with respect to $\sigma_{i,j}$.

By differentiating we can also show that

Lemma 5. *$p_i^*\lambda_i$ is a strictly decreasing function in $\sigma_{i,j}$ if $\sigma_{k,j} = 1$ where $i, k \in \{1, 2\}$ and $i \neq k$.*

Thus, from Lemma 4 $\sigma_{i,j} = 1$ is a best response if $\sigma_{k,j} < 1$. On the other hand, if $\sigma_{i,j} = 1$, the best response is the minimum possible $\sigma_{k,j}$. Note that the minimum possible $\sigma_{k,j}$ is the one for which $\Pi_k = 0$. Note that if $\sigma_{k,j} = 0$, the profit of SA k is zero. Hence, $\sigma_{k,j} = 0$.

[10]The careful reader will note that in obtaining the best response, we implicitly assumed that in any deviation both SAs continued to serve customers so that (26) still applies. The complete proof requires one to also argue that there is no loss in doing this, i.e., any profitable deviation that results in one of the SAs serving no customers is also accounted for.

Now, from Corollary 4, $\Pi_k = 0$ when $\sigma_{i,j} = 1$ is given by

$$\begin{aligned} \sigma_{i,j}\sigma_{k,j}(q_jv - q_j\Lambda/W) &= p_k^* \\ \sigma_{k,j}(q_jv - q_j\Lambda/W) &= q_j(1-\sigma_{k,j})(2\Lambda/(3W) - v/3) \\ \sigma_{k,j} &= \frac{2\Lambda - v}{2v - \Lambda} \end{aligned} \tag{28}$$

Since $\sigma_{k,j} \geq 0$, thus, the best response $\sigma_{k,j}$ is given in (28).

Note that by the construction this is the only possible solution.

Now SA k follows the strategy only if the profit is non-negative. Hence, the strategy where $\sigma_{k,j}$ constitutes a NE is given by the condition in Case (1) of Theorem 3.

If the condition in case (1) is not satisfied, there is no other possible equilibrium where $\sigma_{k,j} > 0$. Hence, SA i will have monopoly power.