Contracts as Entry Barriers for Unlicensed Spectrum

Yining Zhu, Randall A. Berry
Dept. of EECS, Northwestern University, Evanston, IL 60208
yiningzhu2015@u.northwestern.edu, rberry@eecs.northwestern.edu

***Abstract*—Greater unlicensed access to spectrum has the potential to increase competition among wireless service providers and encourage innovations by lowering barriers to entry. However, early providers offering service in such a band could create new entry barriers through the use of contracts that impose a penalty on customers who switch to a new provider. This paper discusses cases in which an exclusive or non-exclusive contract may be signed before entrants come into an unlicensed spectrum market. Our results indicate that the incumbents will always offer an exclusive contract, which would increase the expected customer surplus. The expected social welfare may increase or decrease depending on how we model the customers' demand, and the technology of the incumbents and entrants.**

I. Introduction

Due to the increasing demand for wireless data services, there have been many policy discussions about the efficient use of spectrum. A key distinction is between spectrum being licensed or unlicensed. Licensed Spectrum is exclusively available to the license holder, while unlicensed spectrum can be shared by multiple users provided they follow established technical rules. There have been efforts to expand unlicensed access including the TV white spaces [1] and the Generalized Authorized Access (GAA) tier in the 3.5GHz band [12]. A prime motivation for this is that unlicensed access lowers the barriers to entry and so can potentially lead to more competition and innovation.

This lowering of entry barriers may also have potential drawbacks such as increased congestion in the band and lowering the profits of firms [5], [7]. Though a license is not required to access such a band, early providers using the band can create other forms of entry barriers, such as by using customer contracts, a common practice in the wireless market. Indeed, economists have long recognized that firms may use contracts as a way of impeding entry into a market, and that the use of such contracts may result in lower overall welfare [2]. However, spectrum differs from the commodity market studied in [2] in that spectrum is a congestible resource, so that the Quality of Service (QoS) experienced by users will degrade as more users are served in a band. Contracts that reduce entry could potentially be helpful in reducing congestion and so it is not clear if the conclusions of [2] hold in this setting. In this work, we explore these effects.

In related work, [10] considers contract-based cooperative spectrum sharing between spectrum users, while our paper focuses on contracts between customers and incumbents. A number of papers including [3]-[8] have considered various models for competition among wireless service providers, but have not considered the use of long-term contracts.

This research was supported in part by NSF grants TWC-1314620, AST-1343381 and AST-1547328.

Our approach is based on using the model for contracts from [2] and combining this with a model as in [5], [7] for competition among service providers (SPs) with unlicensed spectrum. As in [5], [7], we assume that customers choose a SP based on the *delivered price*, which is the sum of a service price and a congestion cost. The congestion cost is increasing in the number of customers served in the band modeling the congestible nature of this resource. As in [2], we consider a simplified contract consisting of only two variables: the service price and the *liquid damage*, which is the price a customer pays for breaking the contract.

We assume that there is one SP who is the first to offer service using a new unlicensed band. We refer to this first SP as the incumbent. This SP may offer a long-term contract to its customers as well as give them the opportunity to buy service without a contract. Subsequently, one or more new entrant SPs may also offer service in this band. When such entrants arrive, the customers under contract have the option of breaking the contract and buying service from the new entrant. At the time that customers are offered the contract, we assume that both the customers and the incumbent are uncertain about the future QoS that will be offered by an entrant. Here, we discuss two types of contracts, nonexclusive and exclusive contracts. A *nonexclusive contract* is one in which a customer can purchase service from multiple SPs simultaneously, and an exclusive contract is one in which a customer must first break a contract before getting service from another SP. In the former case, the liquid damage will always be smaller than the contract service price, while in the latter it will exceed the service price. An exclusive contract is signed when customers buy locked devices or get other benefits in addition to the service. If not, there is no reason to set liquid damage greater than the service price, since the customer can always maintain his/her contract and seek service with higher quality. We will show that these two types of contracts have different effects on the unlicensed spectrum market.

The rest of this paper is organized as follows. We first formally introduce our model in Section II as a Bayesian game, where there is uncertainty about the type of the entrants. This uncertainty is with regard to the entrant's congestion function, which, for example, may depend on the technology it uses. We then analyze the game for the case of a single entrant, when the entrant's type distribution is known to both the customers and the incumbent in Section III, and analyze the equilibrium. Then in Section IV, we compare the equilibrium to that in markets where the spectrum is licensed and to markets where no contracts are used. In Section V, we consider cases with multiple entrants. Finally, we conclude in Section VI.

Our work shows that only exclusive contracts can create a barrier to entry. Signing such a contract can always improve customer surplus if customers have the same information about the entrants as the incumbent. However, the effect of contracts on social welfare may be positive or negative. It depends on the customers' demand, the incumbent's technology and the entrants' potential technology.

II. Competition Model

In this paper, we consider a two-period model. In the first period, there is one incumbent SP (SP1) offering service in a given band of unlicensed spectrum in a given area. In the second period there are n potential entrant SPs, denoted SPi, $i = 2, ..., n+1$, who may enter the market in the same area and also offer service in this band. Initially, we assume that $n = 1$ and subsequently generalize to $n > 1$ in Section V.

As in [5], [7], we assume that all SPs in the market compete for a common pool of customers, modeled as non-atomic users with a total mass of 1. The service quality offered by each SPi is characterized via a *congestion cost*, $g_i(x_T)$, which is increasing in the total number of customers, x_T, receiving service from any SP in the band; this models the shared nature of the unlicensed band. For this paper, to simplify our analysis, we assume the congestion costs are linear, i.e., $g_i(x_T) = k_i x_T$. Here, $k_i > 0$ determines the slope of SPi's congestion cost function and may vary across SPs to model differences in the technology they use and the amount of infrastructure they invest in. Note that a smaller slope indicates less congestion and thus a SP with a better service.

We will assume that the incumbent SP's congestion parameter k_1 is fixed in stage 1 and known to all SPs and customers. However, for an entrant, SPi, we assume that in stage 1, its congestion cost is not known to either the other SPs or to the customers. Only at the start of stage 2, when this SP enters the market, does this parameter become known. Further we assume that it is drawn uniformly from $[(1-\alpha)k_1, (1+\alpha)k_1]$, where α is a parameter which controls the range of k_i. We assume that this prior distribution is common knowledge for all SPs and customers. Note that this means that the expected value of k_i is equal to k_1, and with probability $1/2$, k_i is less than k_1, i.e., this is the probability that the entrant has a better service quality than the incumbent.

The customers are characterized by an inverse demand $D(x)$ that represents the value the xth customer places on the service. This is assumed to be a continuous and non-increasing function of $x \in [0, 1]$. In the following sections we will place further restrictions on this to facilitate our analysis. If the xth customer received service from SPi at a price of p_i, then that customer's pay-off is given by the difference between $D(x)$ and its *delivered price* given by $p_i + g_i(x_T)$. Customer's will only accept service (and pay p_i) if this pay-off is positive and if so, they will seek the provider that gives them the largest pay-off.

To begin, consider the competition between one incumbent and one entrant SP in stage 2, when there are no contracts (this is similar to the model studied in [5], [7]). In this case, we assume that SPs compete on price, i.e., each SP announces a price p_i and seeks to maximize its revenue $\pi_i = p_i x_i$, where x_i is the number of customers that accept SPi's service. Given these prices, since customers are non-atomic and seek to maximize their own pay-off, it follows that they must be in a Wardrop equilibrium [9]. In other words, the delivered price of each SP serving customers must be equal and no greater than $D(x_T)$, while the delivered price for any SP not serving customers must be greater than or equal to $D(x_T)$ (here $x_T = x_1 + x_2$). As shown in [8], if $k_2 \neq k_1$ (which occurs with probability one), the outcome of this competition is such that the SP with the better service quality is the only SP that can serve customers at a positive price. Hence, we assume that after seeing its realized congestion parameter k_2, the entrant SP will not choose to enter the market if $k_2 > k_1$; however, the incumbent SP is assumed to not be able to quit the market even if $k_1 < k_2$. This is reasonable, for example, if the entrant would have to pay some cost to offer service while in the incumbent's case this cost is already sunk. Hence, the expected revenue of the incumbent in stage 2 is given by

$$E(\pi_1) = \tfrac{1}{2}\pi_1^M,$$

where π_1^M denotes the incumbent's revenue if it is a monopolist.

Now we consider the case with contracts. The motivation for a contract is for the incumbent to attempt to "lock-in" customers in stage 1 and thus make it less likely for another SP to enter the market in stage 2. As in [2], we consider contracts denoted by $\{P_1^c, P_0\}$, where P_1^c is the service price a customer pays when entering the contract and P_0 represents the damages a customer must pay to break the contract. The contract parameters are determined by the incumbent in an attempt to maximize its expected revenue. The specific timing we consider is as follows:

1) During stage 1, the incumbent offers a contract $\{P_1^c, P_0\}$ to the customers, each of whom may either sign it or not.
2) Next the entrants see their congestion function and decide whether to enter or not.
3) Finally, in stage 3, all SPs in the market compete to maximize their revenue by offering service prices as before.

Note in the last stage, customers who signed a contract may break it and purchase service on the "open market." Also, in this stage, the incumbent may serve customers at the market price that did not sign a contract in stage 1.

An individual customer will sign the contract if her expected pay-off in stage 2 is better than without a contract. These quantities can in turn be determined from the given prior distribution of the entrant's congestion parameter and the assumed profit maximizing behavior of the SPs.

One other property of a contract is whether it is *exclusive* or *non-exclusive*, where by an exclusive contract we mean that customers must break the contract before accepting service from another SP. On the other hand, with a non-exclusive contract, a customer can purchase service from another SP, while still staying in the contract. In the non-exclusive case, note that P_0 cannot be greater than P_1^c. If P_0 is less than P_1^c, then a customer would never break the contract and would simply keep it and seek service from the other SP at the same time. Indeed, non-exclusive contracts were the type studied in [2] (without congestion). However, if the contract is exclusive, then P_0 can exceed P_1^c. Such a contract can be a reasonable model for wireless service where other services or devices are bundled with the service. In this case switching service incurs a greater cost than the service cost. For our assumed setting, we have the following result.

Theorem 1. *Given a general decreasing concave inverse demand function and an increasing convex congestion cost function, nonexclusive contracts offer no barrier to entry and do not improve the expected profit of the incumbent.*

Recall that without contracts, the entrant SP will only enter the market if it has a better congestion cost than SP1, and in this case the entrant will charge a small enough price so that no customers are served by SP1. With a non-exclusive contract $\{P_1^C, P_0\}$, also recall that $P_1^c - P_0 > 0$. Suppose that such a contract was signed by customers in the market and that the entrant has a better congestion cost than SP1. We argue in this case that the entrant must again be able to enter the market and serve all the customers at a positive profit. In particular, for customers that signed the contract, suppose their delivered price with the contract is $P_1^c + g_1$, where g_1 denotes the congestion cost seen by the incumbent's customers. Note the incumbent can choose a price $p_2 < P_1^c - P_0$ and by assumption, the congestion seen by the entrant's customers (g_2) is lower than that of the incumbent (i.e., $g_2 < g_1$). It follows that even accounting for the penalty P_0, the customers will have a lower delivered price with the entrant. Hence, non-exclusive contracts do not provide an entry barrier to the entrant SP. The proof regarding the expected profit of SP1 is omitted due to space considerations. We note that in [2], non-exclusive contracts did improve the incumbent profits, showing a key difference between a model with and without congestion. Given this result, we focus on exclusive contracts in the following.

III. Equilibrium Analysis

In this section we analyze the equilibrium of the model in the previous section for two different forms of the inverse demand.

A. Homogeneous inverse-demand

We first assume that customers have a homogeneous inverse demand, which means all customers have the same reservation price normalized as 1. As shown in Fig. 1, $D(x)$ has a 'box' shape [3] . When SP1 exclusively uses this band, it will maximize its revenue by adjusting its service price p_1 to solve the following optimization:

$$\begin{aligned} &\max\ x_T p_1 \\ &\text{subject to } 0 \le x_T \le 1 \\ &\qquad k_1 x_T + p_1 = 1. \end{aligned}$$

It can be seen that the solution to this is for SP1 to set $p_1 = 1 - k_1$ when $k_1 \le \frac{1}{2}$ and $p_1 = \frac{1}{2}$ when $k_1 > \frac{1}{2}$, so that $x_T = min\{1, \frac{1}{2k_1}\}$. Next, consider the case when SP2 enters

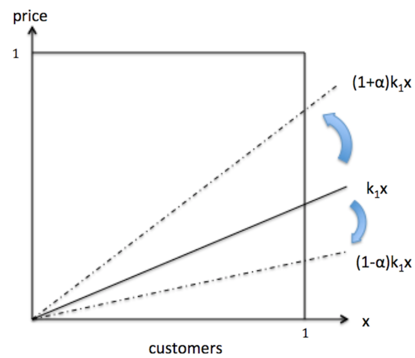

Fig. 1. An example illustrating the case with homogeneous inverse demand.

the market. When $k_1 < \frac{1}{2}$, all customers will be served no matter by which SP. The reason is that if $k_2 < k_1$, SP2 will serve those who didn't sign. Otherwise, SP1 will still serve all the customers to maximize its revenue. Using this we get the following lemma:

Lemma 1. *When $k_1 < \frac{1}{2}$, all customers in the market will sign the contract $\{\frac{1-k_1}{2}, \frac{\alpha k_1}{2} + \frac{1-k_1}{2}\}$ with SP1.*

Proof. A customer will only sign a contract if her expected pay-off is larger than that when she does not sign. If she signs a contract, her expected pay-off must be at least $1 - k_1 - P_1^c$, where this is the pay-off of a customer in the secondary period if it signed the contract and did not break it (here, we are using the fact that all customers are served so that the congestion cost is simply k_1). This follows since if the customer breaks the contract, SP2 must be giving it a pay-off greater than this. If she doesn't sign, then if $k_2 < k_1$, the entrant will enter and compete the delivered price down to k_1 (the value at which p_1 must be zero); otherwise, the entrant will not enter and SP1 will charge the monopolist price of $1 - k_1$, resulting in the customer getting a pay-off of zero. It follows that the customer will sign the contract only if:

$$1 - k_1 - P_1^c \ge \frac{1 - k_1}{2}. \tag{1}$$

Since every customer is homogeneous, they will all sign when this condition is met. Next, to fully specify the contract, SP1 maximizes its expected revenue given by:

$$(1 - \Phi)P_1^c x_0 + \frac{1}{2} p_1 (x_1 - x_0) + \Phi P_0 x_0, \tag{2}$$

where x_0 is number of customers that sign the contract, x_1 is total number of customers SP1 will serve if SP2 does not enter, and p_1 is SP1's price for the new customers that did not sign. Here, $x_1 = x_0$, and p_1 does not exist. Φ is the probability that customers in the contract will switch service, and when $k_1 < 1$,

$$\Phi = Pr\{k_2 < k_1 + P_1^c - P_0\} = \frac{1}{2} - \frac{P_0 - P_1^c}{2\alpha k_1}.$$

Combining this with (2), we have

$$E(\pi_1) = P_1^c + \Phi(P_0 - P_1^c) \quad (3)$$

$$= P_1^c + (\frac{1}{2} - \frac{P_0 - P_1^c}{2\alpha k_1})(P_0 - P_1^c). \quad (4)$$

Considering $P_0 - P_1^c$ as a variable, $E(\pi_1)$ is maximized when

$$P_1^c = \frac{1-k_1}{2} \quad (5)$$

$$P_0 - P_1^c = \frac{\alpha k_1}{2}. \quad (6)$$

Further, we have that

$$E(\pi_1|\text{Contract}) = \frac{1-k_1}{2} + \frac{\alpha k_1}{8} \geq E(\pi_1|\text{No Contract}) = \frac{1-k_1}{2}.$$

So SP1 will offer a contract $\{\frac{1-k_1}{2}, \frac{\alpha k_1}{2} + \frac{1-k_1}{2}\}$ and all customers will sign when $k_1 < \frac{1}{2}$.

□

Lemma 2. *When $\frac{1}{2} < k_1 < 1$, all customers in the market will sign the contract $\{\frac{1-k_1}{2}, \frac{\alpha k_1}{2} + \frac{1-k_1}{2}\}$ with SP1 if $k_1 \leq \frac{1}{2-\frac{\alpha}{2}}$, and $\frac{\alpha}{2}$ customers will sign the contract $\{\frac{1}{4} - \frac{\alpha k_1}{8}, \frac{1}{4} + \frac{3\alpha k_1}{8}\}$ if $k_1 > \frac{1}{2-\frac{\alpha}{2}}$.*

Proof. For this range of k_1, it need not be the case that all customers are eventually served. In particular, if SP2 does not enter the market, then SP1 may not serve all customers (if SP2 does enter, then as before due to the ensuing price competition, all customers will be served). In this case, SP1 seeks to serve $x_T - x_0$ new customers so as to maximize:

$$(x_T - x_0)(1 - k_1 x_T).$$

This is maximized when $x_T = \frac{1}{2k_1} + \frac{x_0}{2}$.

In particular, if $x_T = \frac{1}{2k_1} + \frac{x_0}{2} < 1$, SP1 will not serve all customers in stage 3, when SP2 chooses not to enter. We assume this to be true in the following analysis. Otherwise, all customers are expected to be served in stage 3, which is the same as the former case when $k_1 < \frac{1}{2}$.

The expected payoff of customers who sign the contract is

$$E(Z(x)|\text{Sign}) = \frac{1}{2}[1 - (k_1(\frac{1}{2k_1} + \frac{x_0}{2}) + P_1^c)] + \frac{1}{2}[1 - (k_1 + P_1^c)].$$

This should be at least expected payoff of customers who do not sign which is:

$$E(Z(x)|\text{Not Sign}) = E(Z(x)|\text{No Contract}) = \frac{1-k_1}{2}.$$

From (2) and the condition above, the expected revenue of SP1 is

$$\frac{(1-k_1x_0)^2}{8k_1} + \Phi x_0 P_0 + (1-\Phi)x_0 P_1^c. \quad (7)$$

This is maximized when $P_0 - P_1^c = \frac{\alpha k_1}{2}$. So $x_0 = \frac{\alpha}{2}$, $P_1^c = \frac{1}{4} - \frac{\alpha k_1}{8}$, $P_0 = \frac{1}{4} + \frac{3\alpha k_1}{8}$. Hence,

$$E(\pi_1|\text{Contract}) = \frac{1}{8k_1} + \frac{\alpha^2}{32}k_1 \geq E(\pi_1|\text{No Contract}).$$

So, SP1 will offer this contract. Finally note that the assumed condition $(2 - x_0)k_1 > 1$ is satisfied provided that $k_1 > \frac{1}{2-\frac{\alpha}{2}}$.

If $k_1 \leq \frac{1}{2-\frac{\alpha}{2}}$, all customers will be served (can be verified by analyzing (7)). This is the same case as when $k_1 < \frac{1}{2}$. □

Lemma 3. *When $k_1 \geq 1$, and $\alpha < \frac{5-\sqrt{17}}{2}$, a contract $\{\frac{2-\alpha}{8}, \frac{1}{4} + \frac{3\alpha}{8}\}$ will be signed by $\frac{\alpha}{2k_1}$ customers.*

Proof. We know that SP2 is willing to serve customers who signed with SP1 as long as it can make profit, since SP2 will suffer the congestion from those customers, regardless. However, SP2 might lower its price and make customers who signed suffer from more congestion, so that SP2 can increase the price offered to these customers. This will make the analysis complicated. To avoid this difficulty, we assume that $\frac{1}{2k_2} + \frac{x_0}{2} \leq \frac{1}{k_1}$ always holds, such that the number of customers in service would be $\frac{1}{k_1}$ when SP2 enters (not changing with k_2). We then verify that when $\alpha < \frac{5-\sqrt{17}}{2}$, this assumption is correct. Under these assumptions, SP2 will not join the market if $k_2 \geq k_1$. If $k_2 < k_1$, x_T will be $\frac{1}{k_1}$, so that customers who signed the contract will incur a negative payoff of P_1^c and joining SP2 will not change this situation. Hence, the P_1^c must be chosen to satisfy:

$$\frac{1}{2}(1 - k_1(\frac{x_0}{2} + \frac{1}{2k_1}) - P_1^c) - \frac{1}{2}P_1^c \geq 0.$$

Similar as before, maximizing $E(\pi_1)$ under the above constraints gives the optimal contract $\{\frac{2-\alpha}{8}, \frac{1}{4} + \frac{3\alpha}{8}\}$ signed by $\frac{\alpha}{2k_1}$ customers.

We can verify that when $\alpha < \frac{5-\sqrt{17}}{2}$, the assumption $\frac{1}{2k_2} + \frac{x_0}{2} \leq \frac{1}{k_1}$ holds. □

In summary,

- if $k_1 \leq \frac{1}{2-\frac{\alpha}{2}}$, a contract $\{\frac{1-k_1}{2}, \frac{\alpha k_1}{2} + \frac{1-k_1}{2}\}$ will be signed for all customers;
- if $\frac{1}{2-\frac{\alpha}{2}} < k_1 < 1$, a contract $\{\frac{1}{4} - \frac{\alpha k_1}{8}, \frac{1}{4} + \frac{3\alpha k_1}{8}\}$ will be signed by $\frac{\alpha}{2}$ customers;
- if $k_1 \geq 1$ and $\alpha < \frac{5-\sqrt{17}}{2}$, a contract $\{\frac{2-\alpha}{8}, \frac{1}{4} + \frac{3\alpha}{8}\}$ will be signed by $\frac{\alpha}{2k_1}$ customers.

B. linear inverse-demand/linear congestion

In this section, we discuss a case where customers have heterogeneous demands, i.e., all customers' reservation prices are not the same. We model their reservation price as an inverse demand function $D(x) = 1 - x$ as in Fig. 2. Here we also assume that $\frac{x_0}{2}+\frac{1}{2(1+k_2)} \leq \frac{1}{1+k_1}$ for all possible k_2. This will keep the number of customers in service fixed at $\frac{1}{1+k_1}$ when $k_2 < k_1$ (i.e., this number does not change with k_2) and make the analysis easier. After we solve the equilibrium, this assumption will give us an upper bound on α, which means SP2's technology cannot be too different from SP1.

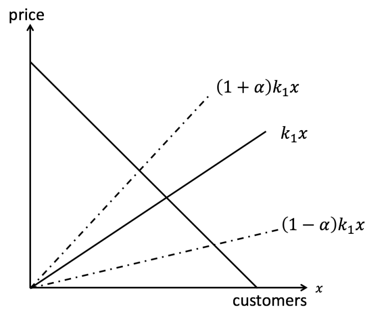

Fig. 2. An example illustrating the case with linear inverse demand.

If $k_2 > k_1$, SP2 will not enter. If x_0 customers have signed the contract with SP1, SP1 will continue to attract new customers who didn't sign before with another service price to maximize its revenue. Similar to the homogeneous case, the total number of customers x_T will be $\frac{x_0}{2}+\frac{1}{2(1+k_1)}$, and the price for new customers is $\frac{1-k_1x_0-x_0}{2}$. If $k_2 < k_1$, SP1 and SP2 will compete until SP1 can no longer lower its price. Therefore, $x_T = \frac{1}{1+k_1}$. Note that it is not possible for SP2 to choose a lower price and attract more customers because of the assumption $\frac{x_0}{2}+\frac{1}{2(1+k_2)} \leq \frac{1}{1+k_1}$. So, the expected number of customers is

$$E(x_T) = \frac{1}{4}x_0 + \frac{3}{4(1+k_1)}. \tag{8}$$

Customers will sign the contract only if their expected payoff is higher. In other words, the expected delivered price for not signing is higher than from signing, i.e.,

$$E(1-x_T) \geq E(k_1x_T + P_1^c).$$

This can be simplified as

$$\frac{x_0}{4} + \frac{1}{4}x_0k_1 + P_1^c \leq \frac{1}{4}. \tag{9}$$

The inequality in (9) should be tight as if not, then more customers would want to sign the contract. SP2 can make customers break the contract when

$$\frac{k_1-k_2}{k_1+1} + P_1^c \geq P_0. \tag{10}$$

Let $y = P_0 - P_1^c$. The contract is broken when

$$k_2 < (1-y)k_1 - y. \tag{11}$$

Similar to the homogeneous case, the expected revenue of SP1 is given by

$$\begin{aligned}E(\pi_1) =& \frac{1}{2}\Big(x_0P_1^c + \frac{1-k_1x_0-x_0}{2(1+k_1)}\cdot\frac{1-k_1x_0-x_0}{2}\Big)\\ &+ \frac{(1+k_1)y}{2\alpha k_1}x_0P_1^c + \frac{(\alpha-y)k_1-y}{2\alpha k_1}x_0(y+P_1^c).\end{aligned}$$

It is maximized when a contract $\{\frac{1}{4}-\frac{\alpha k_1}{8(1+k_1)}, \frac{1}{4}+\frac{3\alpha k_1}{8(1+k_1)}\}$ is offered, and $\frac{\alpha k_1}{2(1+k_1)^2}$ customers sign. This result requires the assumption that $\alpha < (5-\sqrt{17})\frac{1+k_1}{2k_1}$.

IV. Welfare Analysis

In this section, we consider how contracts affect customer surplus, C and overall social welfare, SW. Customer surplus is the sum of all customers' payoffs, while social welfare is the sum of customer surplus and the revenue of all SPs. We first give the relationship among the expected customer surplus in the following cases: SP1 exclusively uses this band (Licensed), the market is unlicensed without contract (No Contract), and the market is unlicensed with contract (Contract). Let C_L, C_{NC} and C_C be the expected customer surplus in these three cases, respectively.

Theorem 2. *For a general decreasing concave demand and increasing convex congestion cost, the customer surplus, C satisfies: $C_L \leq C_{NC} \leq C_C$.*

Proof. $C_L \leq C_{NC}$ is clear since there is a possibility that SP2 will enter and may increase the customers' payoffs. So customer surplus is at least non-decreasing. The remainder of the proof will use the following lemma, whose proof we omit due to space limitations:

Lemma 4. *If $x_0 \geq 0$ customers signed contract with SP1 and SP2 doesn't enter the market, then let x' be the number of customers SP1 will serve. If $x_0 = 0$, let x^* be the number of customers SP1 will serve. Then $x' \geq x^*$.*

We consider two classes of customers, those who signed the contract and those who didn't. For those who signed the contract, they will always get a higher expected payoff compared to not signing because of the optimization constraint. Let x' and x^* be defined as in Lemma 4. Note that those customers who did not sign get higher payoffs than with no contract because $x' \geq x^*$, and $D(x)$ is decreasing. For customer x, his payoff is $\max\{D(x)-D(x^*), 0\}$ when there is no contract, and $\max\{D(x)-D(x'), 0\}$ when there is a contract. Hence,

$$\max\{D(x)-D(x'),0\} \geq \max\{D(x)-D(x^*),0\}. \tag{12}$$

As their payoffs are the same when SP2 enters the market, they have higher expected payoff overall. Since every customer's expected payoffs is higher, customer surplus is increased when contracts are used. $\square$

We next consider social welfare. It can be analyzed in the two special models we studied in Section III and calculated by using the equilibrium results in that section. Likewise, we define SW_L, SW_{NC} and SW_C to be the expected social welfare of the three cases discussed. Here, due to space considerations, we omit the expressions and instead summarize some properties of welfare in different regimes along with some numerical plots illustrating these.

A. *homogeneous inverse-demand/linear congestion*

Under homogeneous inverse-demand, linear congestion functions and a uniform k_2 distribution assumptions, SW has the following properties:

1) SW_C is always less than or equal to SW_{NC}. Hence, in this case, contracts reduce social welfare.
2) When $k_1 \leq \frac{1}{2-\frac{\alpha}{2}}$, $SW_L \leq SW_C \leq SW_{NC}$. So when the spectrum is not scarce and SP1 can use it relatively efficiently, an unlicensed spectrum market with no contracts gives the highest social welfare.
3) However, as k_1 becomes bigger, which means this unlicensed band becomes limited and SP1 uses it inefficiently, licensed access may have higher social welfare than unlicensed access. This is similar to Braesss paradox [11]. When SP1 and SP2 compete, they might lower the price and serve more customers, which increases the congestion, leading to the tragedy of the commons.
4) It is possible that making the spectrum unlicensed improves the expected social welfare when there is no contract applied, while deceases the expected social welfare if we consider contracts in the market.
5) When the uncertainty of entrants (i.e., α) grows, unlicensed access tends to be better than licensed access.

These properties are illustrated in Fig. 3.

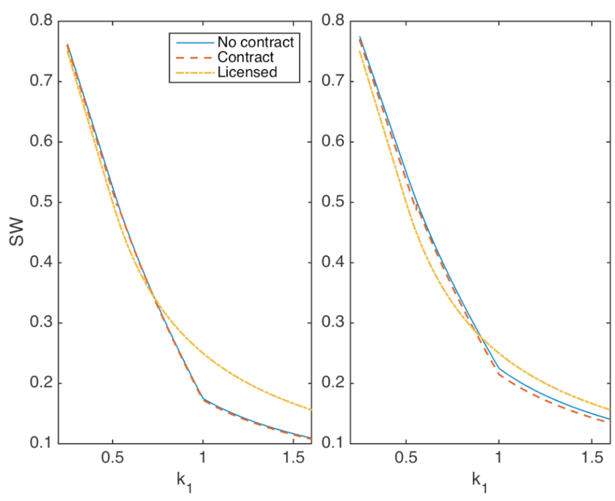

Fig. 3. Two examples of the expected social welfare with homogeneous inverse-demand when $\alpha = 0.2$ (left) and $\alpha = 0.4$ (right).

B. *linear inverse-demand/linear congestion*

For the case of linear inverse demand, the expected social welfare can increase or decrease due to contracts. By directly calculating these quantities we have the following insights:

1) When $k_1 \leq k^* = \frac{1+\sqrt{65}}{8} \approx 1.13$, $SW_{NC} \leq SW_C$ as is shown in Fig. 4. Since SP1 has good technology, the positive effect of contracts resulting in more customers being served when SP2 doesn't enter the market is dominating. Hence, in this case, contracts improve expected social welfare.
2) When $k_1 > k^*$ and α is small, the positive effect of a contract is still dominating, as is shown in Fig. 5.
3) When $k_1 > k^*$ and α is big enough, SW_{NC} will exceed SW_C, since the contract's negative effect of creating barriers against SP2's potential efficient utilization of the spectrum overtakes the effect that more customers are served when SP2 doesn't enter the market. Hence, in this case, contracts reduce social welfare. The intuition here is that as α grows, SP2 has the possibility to contribute to the society better. However, contracts lower this possibility and thus decrease the expected social welfare.

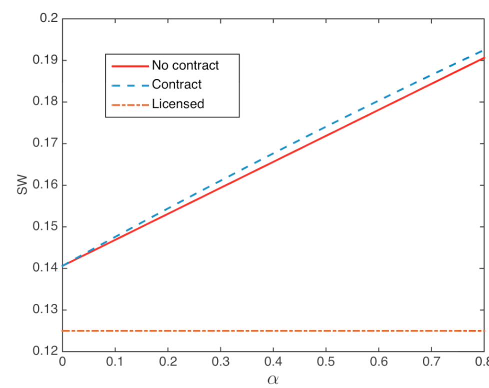

Fig. 4. An example of expected social welfare with linear inverse-demand when $k_1 = 1$.

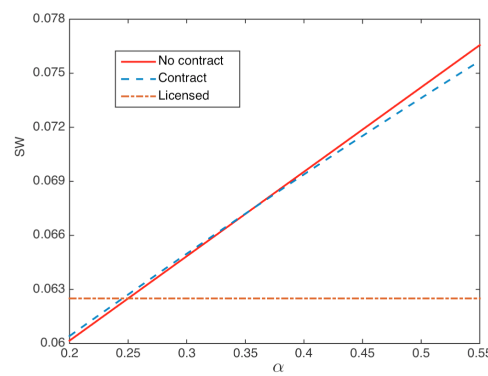

Fig. 5. An example of expected social welfare with linear inverse-demand when $k_1 = 3$.

V. MULTIPLE ENTRANTS

In this section, we discuss the case where an incumbent is facing $n \geq 2$ entrants. We assume that only the entrant who can make a profit when it competes with all the other SPs will enter the market. So only the entrant with the best technology has potential to enter. k_i is uniformly and i.i.d distributed in

$[(1-\alpha)k_1, (1+\alpha)k_1]$. Let $k_{min} = min\{k_2, k_3 \cdots, k_{n+1}\}$, then

$$F_{k_{min}}(z) = 1 - (\frac{(1+\alpha)k_1 - z}{2\alpha k_1})^n. \quad (13)$$

Doing a similar analysis as before, we find that as there are more entrants, more (or same) number of customers will be willing to sign contracts, and SP1 will set a lower contract price but make $P_0 - P_1^c$ larger. Also, the probability of breaking the contract for a customer is increasing with n.

VI. Conclusion

We analyzed a simple model for the use of contracts by incumbent operators in unlicensed spectrum. Unlike the licensed spectrum market, an incumbent SP in the unlicensed market will only be willing to offer exclusive contracts, and under our assumptions, expected customer surplus is increased when such contracts are used, but not necessarily the overall expected social welfare. This work gives us some intuition of how contracts would work in an unlicensed spectrum market and based on this, we could extend this work in several ways including exploring different information assumptions and considering models in which entrants can improve their efficiency by investment.

References

[1] Federal Communications Commission, "Unlicensed Operation in the TV Broadcast Bands/Additional Spectrum for Unlicensed Devices Below 900 MHz and in the 3 GHz Band", FCC Report and Order, September, 2010

[2] P. Aghion and P. Bolton, "Contracts as a Barrier to Entry," *The American economic review*, pp. 388-401, 1987.

[3] C. Liu and R. Berry, "Competition with shared spectrum," *2014 IEEE International Symposium on Dynamic Spectrum Access Networks (DYSPAN)*, 2014.

[4] F. Zhang, and W. Zhang, "Competition between wireless service providers: Pricing, equilibrium and efficiency," *2013 11th International Symposium on Modeling & Optimization in Mobile, Ad Hoc & Wireless Networks (WiOpt)*, 2013.

[5] T. Nguyen, H. Zhou, R. Berry, M. Honig, and R. Vohra, "The Cost of Free Spectrum.," *Operations Research*, vol. 64, no. 6, pp. 1217-1229, 2017.

[6] P. Maill, B. Tuffin, and J. Vigne, "Competition between wireless service providers sharing a radio resource," *International Conference on Research in Networking*, 2012.

[7] T. Nguyen, H. Zhou, R. Berry, M. Honig, and R. Vohra, "The impact of additional unlicensed spectrum on wireless services competition," *2011 IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, 2011.

[8] H. Zhou, R. Berry, M. Honig and R. Vohra, "Investment and competition in unlicensed spectrum." *2012 46th Annual Conference on Information Sciences and Systems (CISS)*, 2012.

[9] J. G. Wardrop, "Some theoretical aspects of road traffic research," *ICE Proceedings: Engineering Divisions*, vol. 1, no. 4, 1952.

[10] L. Duan, L. Gao, and J. Huang, "Contract-based cooperative spectrum sharing," *2011 IEEE Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, 2011.

[11] D. Braess, "ber ein Paradoxon aus der Verkehrsplanung," *Unternehmensforschung* vol. 12, pp. 258-268, 1968.

[12] Federal Communications Commission, "Amendment of the Commission's Rules with Regard to Commercial Operations in the 3550-3650 MHz Band," Report and order and second further notice of proposed rulemaking, 2015.