Throughput Optimal Control of Cooperative Relay Networks

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***Abstract*— We give a model for cooperative communication in a parallel relay network that includes the stochastic arrival of packets and queueing. For this model we provide a throughput optimal network control policy which stabilizes the network for any arrival rate in its stability region. This policy is a generalization of maximum differential backpressure policies which takes into account the potential cooperative gains in the network.**

I. Introduction

In recent years, motivated chiefly by wireless networking applications, there has been interest in models which jointly address "network layer" issues such as the random generation of traffic, delay, and buffer occupancy, along with traditional "physical layer" issues such as modulation, coding, and channel modeling. In [1], [2], models for multiaccess and broadcast channels taking into account both queueing dynamics as well as information-theoretic capacity regions have been considered. For these models, the network *stability region* is characterized; this is the set of arrival rates for which all queues can be stabilized by a feasible rate and power allocation policy. Furthermore, simple *throughput optimal* resource allocation policies are specified, which stabilize the system for any arrival rates in the stability region, without requiring any *a priori* knowledge of the arrival statistics. Results using similar techniques have been shown in [3], [4] for other (non-information theoretic) physical layer models, where joint power/rate allocation and routing are performed for multi-hop transmission.

A feature of all the above models is that each packet follows a single route from the source to the destination. In particular, this does not incorporate the potential gains from various *cooperative relaying* techniques (e.g. [5], [6], [7], [8], [9], [10]). With such techniques, multiple nodes may cooperate in relaying a packet, essentially forming a distributed antenna array. Cooperative communication has mainly been addressed from the physical layer viewpoint, i.e. by studying the achievable rates or diversity gains of given cooperative schemes. A goal of this paper is to study a model of cooperative communication which incorporates the *stochastic arrival of traffic* and *queueing dynamics* at the various nodes in the network. In the next section, we describe such a model based on the parallel Gaussian relay channel studied in [5], [6]. For this model we characterize the network's stability region and give a throughput optimal network control policy. Compared to the models in [1], [2], [3], [4], a new potential trade-off emerges: in order to exploit cooperative gains, information has to be sent along multiple routes; this in turn temporarily increases the congestion within the network.

[1]This research was supported in part by NSF under grants CCR-0313329 and CCR-0313183, and by ARO under grant DAAD19-03-1-0229.

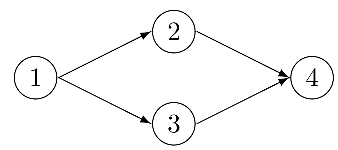

Fig. 1. A four node parallel relay network model.

II. Network Model

Consider a simple network with cooperative communication as shown in Figure 1. This network $\mathcal{G}$ consists of four nodes $\mathcal{V} = \{1, 2, 3, 4\}$. Traffic originates at nodes 1, 2, and 3, and the destination of all traffic in the network is node 4. At the physical layer, we model this as a modified version of a Gaussian parallel relay network [5], [6]. Each node $i \in \mathcal{V}$ has an average power constraint P. Node 1 communicates with nodes 2 and 3 over a Gaussian broadcast channel with bandwidth W. If $X_1(t)$ is the transmitted signal by node 1, then the received signal at node $i = 2, 3$, is given by

$$Y_i(t) = \sqrt{h_{1i}} X_1(t) + Z_i(t), \tag{1}$$

where $Z_i(t)$ is a white Gaussian noise process with noise density $N_0/2$ and h_{1i} is the channel gain between node 1 and i.[1] Nodes 2 and 3 the communicate to node 4 over a 2-user Gaussian multiaccess channel, also with bandwidth W.

[1]Note all channel gains are fixed and known at the transmitters and receivers. Hence, using cooperative transmissions for diversity gains as in [9] is not relevant.

The received signal at node 4 is given by

$$Y_4(t) = \sum_{i=2,3} \sqrt{h_{i4}} X_i(t) + Z_4(t),$$

where $Z_4(t)$ is a white Gaussian noise process with noise density $N_0/2$. All noise processes are mutually independent and independent of the channel inputs. For convenience, we normalize both the bandwidth and the noise power: $W = 1, N_0 W = 1$. We define a *symmetric network* to be one where $h_{12} = h_{13} = 1$ and $h_{24} = h_{34} = 1$.

To simplify our discussion, we assume that at any time the network is either operating in the multi-access mode or the broadcast mode, i.e., if $X_1(t) > 0$ then $X_2(t) = 0$ and $X_3(t) = 0$, and likewise if $X_2(t) > 0$ or $X_3(t) > 0$ then $X_1(t) = 0$. From the point of view of nodes 2 and 3, this enforces a *half-duplexing constraint* [9], [10], i.e. these nodes cannot transmit and receive simultaneously. This is considered a realistic constraint in practical systems. However, we note that our assumption also prohibits schedules which do not violate the half-duplexing constraint, such as node 1 transmitting to node 2, while node 3 transmits to node 4. Also, as in [5], we do not consider direct transmissions from node 1 to node 4. This is reasonable when the distance between these nodes in large. Both of the above-mentioned possibilities could in principle be included in our model at the expense of more complicated notation.

Cooperation in this network is achieved by having nodes 2 and 3 cooperate to relay information from node 1 to node 4. We assume that this is accomplished by having both nodes use a *decode and forward* strategy. Namely, they will both receive and decode the same packet from node 1; eventually, they will simultaneously transmit this packet to node 4 by coherently beamforming the received signal at node 4. For example, in a symmetric network, if node 2 and 3 cooperatively transmit a packet using the maximum power, P, then the received signal power will be $4P$. Of course, this requires that the nodes must be perfectly synchronized. In the absence of perfect synchronization, other cooperative techniques could be used. Also, we note that in general this is not the optimal cooperative strategy from the view of maximizing capacity.[2]

We do not require that all traffic from node 1 to 4 is relayed using this cooperative mode; node 1 can also send "direct" traffic to either node 2 or 3, which the receiving node then individually relays to node 4. Exogenous traffic arrives at node $i = 1, 2, 3$ according to an ergodic counting process $A_i(t)$, where $A_i(t)$ is the number of packet arrivals up to time t. The packet lengths Z_i of exogenous traffic at node i are i.i.d. with $E[Z_i] < \infty$ and $E[Z_i^2] < \infty$. Each node will store all arriving packets in an infinite capacity buffer until they are transmitted. Let $U_1(t)$ be the number of untransmitted bits (unfinished work) at node 1, and let $U_{id}(t)$ and $U_{ic}(t)$ respectively be the unfinished work of direct and cooperative traffic at node $i = 2, 3$. All exogenous arrivals at nodes 2 and 3 are included in the *direct traffic*. By assumption, all cooperative traffic is received at both nodes 2 and 3. Hence, for all t, $U_{2c}(t) = U_{3c}(t)$, and so we will denote both of these quantities by $U_c(t)$. Let $\boldsymbol{U}(t) = (U_1(t), U_c(t), U_{2d}(t), U_{3d}(t))$ denote the joint queue state at time t. We consider the case where given $\boldsymbol{U}(t)$ at time t, a network controller specifies a rate allocation $\boldsymbol{R}(t) = (R_1^c, R_{12}^d, R_{13}^d, R_4^c, R_{24}^d, R_{34}^d)$, where R_{ij}^d is the rate of direct traffic between nodes i and j, R_1^c is the rate node 1 sends cooperative traffic to nodes 2 and 3, and R_4^c is the rate nodes 2 and 3 cooperatively forward traffic to node 4. At times, it will be more convenient to denote the components of $\boldsymbol{R}(t)$ as $(R_1(t), R_2(t), \ldots, R_6(t))$, where for example $R_6 \equiv R_{34}^d$.

The rate allocation chosen for time t must respect the half-duplex constraint described above and given that the network operates in the broadcast or multiaccess mode, the rates must lie in the corresponding capacity region.[3] We describe these capacity regions next. First consider the broadcast mode and without loss of generality assume that $h_{12} \leq h_{13}$. Let $\mathcal{C}_{BC}$ be the capacity region of the two-user Gaussian broadcast channel defined by (1). Then it follows that the rates $(R_1^c, R_{12}^d, R_{13}^d)$ must satisfy $(R_{12}^d + R_1^c, R_{13}^d) \in \mathcal{C}_{BC}$. Let the cooperative broadcast region $\mathcal{C}_{CBC}$ be the set of all such allowable $(R_1^c, R_{12}^d, R_{13}^d)$. For a symmetric network ($h_{12} = h_{13}$) C_{CBC} reduces to the set of non-negative rates that lie in the simplex defined by

$$\sum_{i=1}^{3} R_i \leq \log(1 + P). \tag{2}$$

In the multiaccess mode, if nodes 2 and 3 only send direct traffic ($R_4^c = 0$), then the transmission rates (R_{24}^d, R_{34}^d) must lie in the corresponding multiaccess capacity region $\mathcal{C}_{MAC}$; this is the set of non-negative rates that satisfy

$$\sum_{i \in \mathcal{S}} R_{i4}^d \leq \log\left(1 + \sum_{i \in S} h_{i4} P\right) \ \forall S \subseteq \{2, 3\}.$$

If both nodes send only cooperative traffic ($R_{24}^d = R_{34}^d = 0$) then the transmission rate R_4^c must satisfy

$$R_4^c \leq \log\left(1 + (\sqrt{h_{24}} + \sqrt{h_{34}})^2 P\right).$$

In addition, we allow the nodes to transmit both cooperative and direct traffic simultaneously. One way to model this is to allow time-sharing between the above two modes. More generally, we can view this as a type of 3-user multiaccess channel, with 2 users corresponding to the direct traffic for nodes 2 and 3, respectively, and a third user corresponding to the cooperative traffic.[4] The difference here is that the power constraints of the users are coupled. If both users 2 and 3, devote a fraction $\alpha \in [0, 1]$ of their power to

[2]Indeed, the optimal strategy and the capacity of the parallel Gaussian relay channel is an open problem [6].

[3]It is reasonable to assume that we can achieve any rate in these regions if the times at which we apply the controls are sufficiently separated as to allow the use of long codewords.

[4]A key assumption here is that the encoding of the traffic by these three "users" is done only based on their own message and that the messages are independent.

cooperative traffic, then we assume that they can achieve any rates $(R_4^c, R_{24}^d, R_{34}^d) \equiv (R_4, R_5, R_6)$ which satisfy

$$\sum_{i \in \mathcal{S}} R_i \leq \log\left(1 + \sum_{i \in S} P_i(\alpha)\right) \quad \forall S \subseteq \{4, 5, 6\}, \qquad (3)$$

where $P_4(\alpha) = (\sqrt{h_{24}} + \sqrt{h_{34}})^2 \alpha P$, $P_5(\alpha) = h_{24}(1-\alpha)P$, and $P_6(\alpha) = h_{34}(1-\alpha)P$. Let $\mathcal{C}_{CMAC}(\alpha)$ denote this set of feasible rates for a particular power splitting parameter α. Then we assume that the controller can choose any rates from the cooperative-MAC capacity region given by $\mathcal{C}_{CMAC} = \bigcup_{\alpha \in [0,1]} \mathcal{C}_{CMAC}(\alpha)$. It can be verified that the resulting region is convex.

Let $\overline{\mathcal{C}}_{CBC}$ and $\overline{\mathcal{C}}_{CMAC}$ be $\mathcal{C}_{CBC}$ and $\mathcal{C}_{CMAC}$ embedded in $\mathbb{R}_+^6$, respectively. That is, $(R_1^c, R_{12}^d, R_{13}^d) \in \mathcal{C}_{CBC}$ if and only if $(R_1^c, R_{12}^d, R_{13}^d, 0, 0, 0) \in \overline{\mathcal{C}}_{CBC}$, and $(R_4^c, R_{24}^d, R_{34}^d) \in \mathcal{C}_{CMAC}$ if and only if $(0, 0, 0, R_4^c, R_{24}^d, R_{34}^d) \in \overline{\mathcal{C}}_{CMAC}$. Under the duplex constraint, the overall physical-layer capacity region is $\mathcal{C} = \text{conv}(\overline{\mathcal{C}}_{CBC}, \overline{\mathcal{C}}_{CMAC})$, i.e. the convex hull of these two sets.

III. Network Stability Region and Throughput Optimal Rate Allocation

Given the model in Section II, we proceed to characterize the network stability region and the throughput optimal rate allocation policy. Although the results we obtain here may be reminiscent of results for conventional networks [3], [4], we shall find that the cooperative nature of the parallel relay network introduces some significantly new elements.

A. Stability Region

Let $\lambda_i = \lim_{t \to \infty} A_i(t)/t$ be the packet arrival rate of traffic to node i. Let $\rho_i = \lambda_i \mathsf{E}[Z_i]$ be the corresponding bit arrival rate. We say that queue i is *stable* if $\limsup_{t \to \infty} \frac{1}{t} \int_0^t 1_{[U_i(\tau) > \xi]} d\tau \to 0$ as $\xi \to \infty$, where $1_{\{\cdot\}}$ is the indicator function. The network stability region $\mathcal{S}$ is the closure of the set of all $(\rho_1, \rho_2, \rho_3) \in \mathbb{R}_+^3$ for which there exists some feasible rate allocation policy $\mathcal{R}$ defined by $\boldsymbol{R} = \mathcal{R}(\boldsymbol{u}) \in \mathcal{C}$, where $\boldsymbol{u} = (u_1, u_c, u_{2d}, u_{3d})$, which can guarantee that all queues are stable. Note that $\mathcal{R}$ is a function of the queue state $\boldsymbol{u}$ and can assign any rate from the physical-layer capacity region $\mathcal{C}$.

Using arguments similar to those in [3], [4], we can establish the following.

Theorem 1: The stability region $\mathcal{S}$ of the parallel relay network of Figure 1 is set of all $(\rho_1, \rho_2, \rho_3) \in \mathbb{R}_+^3$ for which there exist non-negative flow variables $f^c, f_{12}^d, f_{13}^d, f_{24}^d, f_{34}^d$ which support (ρ_1, ρ_2, ρ_3) relative to the weighted graph defined by the long-term rates given by $\mathcal{C}$. That is, the following flow conservation relations must be satisfied: $\rho_1 = f^c + f_{12}^d + f_{13}^d$, $\rho_{2d} = f_{24}^d - f_{12}^d$, $\rho_{2d} = f_{34}^d - f_{13}^d$. In addition, $(f^c, f_{12}^d, f_{13}^d, f^c, f_{24}^d, f_{34}^d) \in \mathcal{C}$.

B. Throughput Optimal Rate Allocation

Theorem 1 states that if $\boldsymbol{\rho} = (\rho_1, \rho_2, \rho_3) \in \text{int}(\mathcal{S})$, then the queues can be stabilized. In general, however, this may require knowing the value of $\boldsymbol{\rho}$. In reality, $\boldsymbol{\rho}$ can be learned only over time, and may be variable. One would prefer to find *adaptive* rate allocation policies which can stabilize the network *without* knowing $\boldsymbol{\rho}$, as long as $\boldsymbol{\rho} \in \text{int}(\mathcal{S})$. These rate allocation policies are called *throughput optimal*. As shown in [4], a throughput optimal resource allocation policy for stochastic networks with physical-layer capacity regions turns out to be a generalization of the *maximum differential backlog* (MDB) policy first proposed by Tassiulas [3]. Due to cooperative transmissions, however, the parallel relay network considered here is somewhat different from the networks considered in [4]. Nevertheless, we show that the MDB policy can be adapted to produce a throughput optimal rate allocation policy for the parallel relay network.

We consider examine the evolution of the network at time instants separated by $T > 0$ units of time, where T is sufficiently large to allow for large coding lengths. We make the following assumptions for the packet arrival processes. Let $\boldsymbol{A}_k = (A_{1k}, A_{2k}, A_{3k})$ be the arrival vector for the kth T-slot. We assume $\{\boldsymbol{A}_k : k \in \mathbb{Z}_+\}$ are i.i.d. according to distribution $\pi_{\boldsymbol{A}}$ with mean $\mathsf{E}[\boldsymbol{A}] = \boldsymbol{\lambda} T$, where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3)$ are the arrival rates. Furthermore, assume that for each i, $\mathsf{E}[A_i^2] < \infty$, and $\Pr\left(\bigcap_{i=1}^3 \{A_i = 0\}\right) > 0$. These assumptions on the arrival process, for example, clearly hold for independent homogeneous Poisson arrival processes. The above assumptions can be relaxed to the Markov modulated case.

Theorem 2: A throughput optimal rate allocation policy $\mathcal{R}^*(\boldsymbol{u})$ for the parallel relay network of Figure 1 is given by the solution to the following optimization:

$$\begin{aligned} \max_{\boldsymbol{R} \in \mathcal{C}} \quad & (u_1 - 2u_c)R_1^c + (u_1 - u_{2d})R_{12}^d + (u_1 - u_{3d})R_{13}^d \\ & + u_{2d}R_{24}^d + u_{3d}R_{34}^d + 2u_c R_4^c \end{aligned} \qquad (4)$$

Note that the policy in (4) is the not the same as the conventional MDB policy of [3], [4]. In particular, the coefficient terms $u_1 - 2u_c$ and $2u_c$ reflect the *queue coupling* effect induced by the cooperative transmission structure. We refer to the policy of (4) as the *Cooperative Maximum Differential Backlog* (CMDB) policy.

Proof of Theorem 2: To show that the CMDB policy stabilizes the network for any $\boldsymbol{\rho} = (\rho_1, \rho_2, \rho_3) \in \text{int}(\mathcal{S})$, it is convenient to consider a fictitious network $\mathcal{G}_f$ which is the same as the network $\mathcal{G}$, except that arrivals are allowed to enter the queues $2c$ and $3c$. Let $\mathcal{S}_f$ be the stability region of the network $\mathcal{G}_f$. It is clear that if the CMDB policy stabilizes $\mathcal{G}_f$ for all $(\rho_1, \rho_{2c}, \rho_{2d}, \rho_{3c}, \rho_{3d}) \in \text{int}(\mathcal{S}_f)$ such that $\rho_{2c} = \rho_{3c} = 0$, then CMDB also stabilizes $\mathcal{G}$ for all $\boldsymbol{\rho} = (\rho_1, \rho_2, \rho_3) \in \text{int}(\mathcal{S})$. Therefore, from now on, we concentrate on the artificial network $\mathcal{G}_f$.

To show that the CMDB policy stabilizes $\mathcal{G}_f$ for all $(\rho_1, 0, \rho_{2d}, 0, \rho_{3d}) \in \text{int}(\mathcal{S}_f)$, we use an extension of Foster's Criterion for the convergence of Markov chains [1], [2], [4]. Consider the Lyapunov function $V(\boldsymbol{u}) \equiv u_1^2 + \sum_{i=2}^{3}(u_{ic}^2 + u_{id}^2)$. We wish to show that there exists a compact subset $\Lambda \subset \mathbb{R}_+^5$ such that under the CMDB policy, $\mathsf{E}[V(\boldsymbol{U}(t+T)) - V(\boldsymbol{U}(t))|\boldsymbol{U}(t) = \boldsymbol{u}] < -\epsilon$ for all $\boldsymbol{u} \notin \Lambda$, where $\epsilon > 0$. This, along with some other technical conditions [4], implies the existence of a steady state distribution for $\boldsymbol{U}$.

We have:

$$\begin{aligned}
&U_1^2(t+T) \\
&= [(U_1(t) + B_1(t) - (R_1^c(t) + R_{12}^d(t) + R_{13}^d(t))T)^+]^2 \\
&\leq (U_1(t) + B_1(t) - (R_1^c(t) + R_{12}^d(t) + R_{13}^d(t))T)^2 \\
&\leq U_1^2(t) - 2TU_1(t)\left(R_1^c(t) + R_{12}^d(t) + R_{13}^d(t) - \frac{B_1(t)}{T}\right) \\
&\quad + B_1^2(t) + (R_1^c(t) + R_{12}^d(t) + R_{13}^d(t))^2T^2
\end{aligned}$$

Here $B_1(t)$ is the number of bits arriving to queue 1 in the tth slot, and $(x)^+$ denotes $\max(x, 0)$. Similarly, for $i = 2, 3$,

$$\begin{aligned}
U_{ic}^2(t+T) &\leq U_{ic}^2(t) - 2TU_{ic}(t)(R_4^c(t) - R_1^c(t))) \\
&\quad + (R_1^c(t)^2 + R_4^c(t)^2)T^2, \\
U_{id}^2(t+T) &\leq U_{id}^2(t) - 2TU_{id}(t)(R_{i4}^d(t) - R_{1i}^d(t) \\
&\quad - B_{id}(t)/T) + B_{id}(t)^2 + 2B_{id}(t)R_{1i}^d(t)T \\
&\quad + (R_{1i}^d(t)^2 + R_{i4}^d(t)^2)T^2
\end{aligned}$$

Note that since $\rho_{2c} = \rho_{3c} = 0$, there are no exogenous arrivals to queues $2c$ and $3c$. Taking conditional expected value of both sides of the above inequalities given the event $\boldsymbol{U}(t) = \boldsymbol{u}$, and re-arranging, we have

$$\begin{aligned}
&\mathsf{E}[V(\boldsymbol{U}(t+T)) - V(\boldsymbol{U}(t))|\boldsymbol{U}(t) = \boldsymbol{u}] \\
&\leq -2Tu_1\mathsf{E}\left[R_1^c(t) + R_{12}^d(t) + R_{13}^d(t) - \frac{B_1(t)}{T}|\boldsymbol{U}(t) = \boldsymbol{u}\right] \\
&\quad -2T(2u_c)\mathsf{E}[R_4^c(t) - R_1^c(t)|\boldsymbol{U}(t) = \boldsymbol{u}] \\
&\quad -2T\sum_{i=2}^{3} u_{id}\mathsf{E}\left[R_{i4}^d(t) - R_{1i}^d(t) - \frac{B_{id}(t)}{T}|\boldsymbol{U}(t) = \boldsymbol{u}\right] \\
&\quad +\beta
\end{aligned} \tag{5}$$

where $\beta > 0$ is an upper bound on a sum of terms involving the second moments of the bit arrivals in the tth slot (which are bounded since the second moments of the packet arrivals and the packet sizes are bounded), and powers of transmission rates (which are bounded since $\mathcal{C}$ is bounded).

Let $\mathsf{E}_{\boldsymbol{u}}[X]$ denote $\mathsf{E}[X|\boldsymbol{U}(t) = \boldsymbol{u}]$. Note that

$$\begin{aligned}
&u_1\mathsf{E}_{\boldsymbol{u}}[R_1^c(t) + R_{12}^d(t) + R_{13}^d(t)] + 2u_c\mathsf{E}_{\boldsymbol{u}}[R_4^c(t) - R_1^c(t)] \\
&+ \sum_{i=2}^{3} u_{id}\mathsf{E}_{\boldsymbol{u}}[R_{i4}^d(t) - R_{1i}^d(t)] \\
&= (u_1 - 2u_c)\mathsf{E}_{\boldsymbol{u}}[R_1^c(t)] + (u_1 - u_{2d})\mathsf{E}_{\boldsymbol{u}}[R_{12}^d(t)] \\
&\quad + (u_1 - u_{3d})\mathsf{E}_{\boldsymbol{u}}[R_{13}^d(t)] + u_{2d}\mathsf{E}_{\boldsymbol{u}}[R_{24}^d(t)] \\
&\quad u_{3d}\mathsf{E}_{\boldsymbol{u}}[R_{34}^d(t)] + 2u_c\mathsf{E}_{\boldsymbol{u}}[R_4^c(t)]
\end{aligned} \tag{6}$$

For any $(\rho_1, 0, \rho_{2d}, 0, \rho_{3d}) \in \text{int}(\mathcal{S}_f)$, there exists $\delta > 0$ such that $(\rho_1 + \delta, \delta, \rho_{2d} + \delta, \delta, \rho_{3d} + \delta) \in \mathcal{S}_f$. Therefore, there exist non-negative flow variables $(f_1^c, f_{12}^d, f_{13}^d, f_4^c, f_{24}^d, f_{34}^d) \in \mathcal{C}$ such that $\rho_1 + \delta = f_c^1 + f_{12}^d + f_{13}^d$, $\rho_{2d} + \delta = f_{24}^d - f_{12}^d$, $\rho_{3d} + \delta = f_{34}^d - f_{13}^d$, and $\delta = f_c^4 - f_c^1$. We therefore have

$$\begin{aligned}
&u_1(\rho_1 + \delta) + u_{2d}(\rho_{2d} + \delta) + u_{3d}(\rho_{3d} + \delta) + 2u_c\delta \\
&= u_1(f_c^1 + f_{12}^d + f_{13}^d) + u_{2d}(f_{24}^d - f_{12}^d) + u_{3d}(f_{34}^d - f_{13}^d) \\
&\quad + 2u_c(f_c^4 - f_c^1) \\
&= (u_1 - 2u_c)f_1^c + (u_1 - u_{2d})f_{12}^d + (u_1 - u_{3d})f_{13}^d \\
&\quad + u_{2d}f_{24}^d + u_{3d}f_{34}^d + 2u_cf_4^c
\end{aligned}$$

Let $\boldsymbol{R}(t) = (R_1^c(t), R_{12}^d(t), R_{13}^d(t), R_4^c(t), R_{24}^d(t), R_{34}^d(t))$ be chosen according to the CMDB rule described in (4). Then, since $(f_1^c, f_{12}^d, f_{13}^d, f_4^c, f_{24}^d, f_{34}^d) \in \mathcal{C}$, $u_1(\rho_1 + \delta) + u_{2d}(\rho_{2d} + \delta) + u_{3d}(\rho_{3d} + \delta) + 2u_c\delta$ is less than or equal to the RHS of (6). Since $\mathsf{E}[B_1(t)/T] = \rho_1$ and $\mathsf{E}[B_{id}(t)/T] = \rho_{id}$ for $i = 1, 2$, the RHS of (5) implies

$$\begin{aligned}
&\mathsf{E}[V(\boldsymbol{U}(t+T)) - V(\boldsymbol{U}(t))|\boldsymbol{U}(t) = \boldsymbol{u}] \\
&\quad \leq \beta - 2T\delta(u_1 + u_{2d} + u_{3d} + 2u_c).
\end{aligned}$$

Let $\Lambda = \{\boldsymbol{u} : u_1 + u_{2d} + u_{3d} + 2u_c \leq \frac{\beta+\epsilon}{2T\delta}\}$. Then, for any $\epsilon > 0$, and any $\boldsymbol{u} \notin \Lambda$, $\mathsf{E}[V(\boldsymbol{U}(t+T)) - V(\boldsymbol{U}(t))|\boldsymbol{U}(t) = \boldsymbol{u}] < -\epsilon$.

We have shown that the CMDB policy stabilizes $\mathcal{G}_f$ for all $(\rho_1, 0, \rho_{2d}, 0, \rho_{3d}) \in \text{int}(\mathcal{S}_f)$. Thus, we have also shown that the CMDB policy stabilizes $\mathcal{G}$ for all $\boldsymbol{\rho} = (\rho_1, \rho_2, \rho_3) \in \text{int}(\mathcal{S})$. □

IV. Calculating the CMBD policy

We now focus on solving the optimization problem (4) required by the CMBD policy. We focus on a symmetric network ($h_{12} = h_{13} = h_{24} = h_{34} = 1$) in this section. In this case, $\mathcal{C}_{CBC}$ is given by (2) and the power variables used in the definition of $\mathcal{C}_{CMAC}(\alpha)$ become $P_4(\alpha) = 4\alpha P$, $P_5(\alpha) = P_6(\alpha) = (1 - \alpha)P$.

For simplicity of notation, let $(w_1, w_2, w_3, w_4, w_5, w_6)$ denote $(u_1 - 2u_c,\ u_1 - u_{2d},\ u_1 - u_{3d},\ 2u_c,\ u_{2d},\ u_{3d})$, and let $(R_1, R_2, R_3, R_4, R_5, R_6)$ denote $(R_1^c, R_{12}^d, R_{13}^d, R_4^c, R_{24}^d, R_{34}^d)$. The CMBD policy can now be expressed as

$$\max_{\boldsymbol{R}\in\mathcal{C}} \sum_{i=1}^{6} w_iR_i, \tag{7}$$

Note that the solution $\boldsymbol{R}^*$ to (7) lies in $\text{conv}(\overline{\mathcal{C}}_{CBC}, \overline{\mathcal{C}}_{CMAC}(\alpha^*))$ for some $\alpha^* \in [0, 1]$, where $\overline{\mathcal{C}}_{CMAC}(\alpha^*)$ is the appropriate embedding of $\mathcal{C}_{CMAC}(\alpha^*)$ in $\mathbb{R}_+^6$. Since $\overline{\mathcal{C}}_{CBC}$ and $\overline{\mathcal{C}}_{CMAC}(\alpha^*)$ are both convex polytopes, $\text{conv}(\overline{\mathcal{C}}_{CBC}, \overline{\mathcal{C}}_{CMAC}(\alpha^*))$ is also a convex polytope. Now since $\boldsymbol{R}^*$ also maximizes the *linear* objective of (7) over $\text{conv}(\overline{\mathcal{C}}_{CBC}, \overline{\mathcal{C}}_{CMAC}(\alpha^*))$, $\boldsymbol{R}^*$ is (without loss of optimality) an extreme point of $\text{conv}(\overline{\mathcal{C}}_{CBC}, \overline{\mathcal{C}}_{CMAC}(\alpha^*))$. Thus, $\boldsymbol{R}^*$ is an extreme point either of $\overline{\mathcal{C}}_{CBC}$ or an extreme point of $\overline{\mathcal{C}}_{CMAC}(\alpha^*)$. We now consider these cases separately.

Case 1: $\boldsymbol{R}^*$ is an extreme point of $\overline{\mathcal{C}}_{CBC}$. In this case, $\boldsymbol{R}^*$ has the form $(R_1^*, R_2^*, R_3^*, 0, 0, 0)$. Thus, (R_1^*, R_2^*, R_3^*) solves

$$\max_{\boldsymbol{R} \in \mathcal{C}_{CBC}} \sum_{i=1}^{3} w_i R_i. \tag{8}$$

Since $\mathcal{C}_{CBC}$ is a simplex, (R_1^*, R_2^*, R_3^*) is easily given as follows. Let $w_{[1]} \geq w_{[2]} \geq w_{[3]}$ be w_1, w_2, w_3 arranged in *decreasing* order. Then $R_{[1]}^* = \log(1+P)$ and $R_{[2]}^* = R_{[3]}^* = 0$. Thus, whenever the CMDB policy operates in the broadcast mode, it allocates maximum rate $\log(1+P)$ to the traffic type with the largest weight w_i. The optimal value of (8) is then $L_{CBC}^*(w_1, w_2, w_3) = (\max_{i=1,2,3} w_i) \log(1+P)$. Under the assumption that $\boldsymbol{R}^*$ is an extreme point of $\overline{\mathcal{C}}_{CBC}$, $L_{CBC}^*(w_1, w_2, w_3)$ is equal to the optimal value of (7), $L^*(w_1, \ldots, w_6)$.

Case 2: $\boldsymbol{R}^*$ is an extreme point of $\overline{\mathcal{C}}_{CMAC}(\alpha^*)$. In this case, $\boldsymbol{R}^*$ has the form $(0, 0, 0, R_4^*, R_5^*, R_6^*)$. Thus, (R_4^*, R_5^*, R_6^*) solves

$$\max_{\boldsymbol{R} \in \mathcal{C}_{CMAC}(\alpha^*)} \sum_{i=4}^{6} w_i R_i \tag{9}$$

Since $\mathcal{C}_{CMAC}(\alpha^*)$ is a *polymatroid* [11], (R_4^*, R_5^*, R_6^*) can be explicitly given as follows. Let $w_{[4]}, w_{[5]}, w_{[6]}$ be the largest, second largest, and smallest element of $\{w_4, w_5, w_6\}$, respectively. Then

$$R_{[i]}^* = \log\left(1 + \frac{P_{[i]}(\alpha^*)}{1 + \sum_{j<i} P_{[j]}(\alpha^*)}\right), \quad i = 4, 5, 6. \tag{10}$$

For instance, if $w_4 \geq w_5 \geq w_6$, then $R_4 = \log(1 + 4\alpha^* P)$, $R_5 = \log\left(1 + \frac{(1-\alpha^*)P}{1+4\alpha^* P}\right)$, $R_6 = \log\left(1 + \frac{(1-\alpha^*)P}{1+(3\alpha^*+1)P}\right)$.

Next, to find α^*, we can solve

$$\max_{\alpha \in [0,1]} \sum_{i=4}^{6} w_{[i]} \log\left(1 + \frac{P_{[i]}(\alpha)}{1 + \sum_{j<i} P_{[j]}(\alpha)}\right). \tag{11}$$

Let $L(\alpha)$ be the objective in (11). For the case of $w_4 \geq w_5 \geq w_6$, it can be verified that $L(\alpha)$ is concave in α over $[0, 1]$, and that $L'(\alpha) \geq 0$ for all $\alpha \in [0, 1]$. Thus, $\alpha^* = 1$, i.e. *all power is allocated to cooperative transmission* over the MAC. In other cases, $L(\alpha)$ may not be concave, and one needs to solve for the stationary points of $L(\alpha)$ and compare the value of $L(\alpha)$ at the stationary points with its values on the boundaries.

Let $L_{CMAC}^*(w_4, w_5, w_6)$ be the optimal objective of (11). Under the assumption that $\boldsymbol{R}^*$ is an extreme point of $\overline{\mathcal{C}}_{CMAC}$, we have $L_{CMAC}^*(w_4, w_5, w_6) = L^*(w_1, \ldots, w_6)$.

The previous discussion for Cases 1 and 2 operated under the *assumption* that $\boldsymbol{R}^*$ is an extreme point of $\overline{\mathcal{C}}_{CBC}$ or that $\boldsymbol{R}^*$ is an extreme point of $\overline{\mathcal{C}}_{CMAC}$. Using these arguments, on the other hand, it is easy to see that the following is true:

Theorem 3: Consider the four-node parallel relay network with differential queue backlogs $w_1, w_2, \ldots, w_6$. If $L_{CBC}^*(w_1, w_2, w_3) \geq L_{CMAC}^*(w_4, w_5, w_6))$, then the optimal solution $\boldsymbol{R}^*$ to (7) is $(R_1^*, R_2^*, R_3^*, 0, 0, 0)$, where $R_{[1]}^* = \log(1+P)$ and $R_{[2]}^* = R_{[3]}^* = 0$. Otherwise, $\boldsymbol{R}^* = (0, 0, 0, R_4^*, R_5^*, R_6^*)$, where $R_{[i]}^*$ is given by (10) and α^* is given by (11).

V. Conclusions

In this paper we considered a model of a parallel relay network that incorporates the stochastic arrival of traffic within the network. For this model we showed that a variation of a maximum back pressure policy is throughput optimal, where this policy is modified to incorporate the potential gains of cooperative communication. We only considered one type of cooperation, namely decode and forward combined with beamforming. Potential future directions include considering other types of cooperation as well as other network topologies.

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