Striving for Social Status

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1. INTRODUCTION

Since at least Veblen's (1899) classic work on conspicuous consumption, economists and social scientists have recognized that social comparisons can influence individual decisions. People compare their consumption, their income, and their belongings to those of people around them, and they strive to maintain their position within their community. A myriad of empirical work has demonstrated that "happiness" does not depend on absolute levels of wealth or consumption of a good, but on the context in which this wealth is held [1]. This effect is best described by the following canonical example. Imagine you have the choice between living in two different worlds

—A: Your yearly income is \$50,000 and others earn \$25,000.
—B: Your yearly income is \$100,000 and others earn \$200,000.

If people's concern were over absolute wealth, world B is clearly better for every individual. However, if your concerns are relative, then you may prefer to live in world A. Indeed, various studies have showed that respondents offer favor world A.

We study such social comparisons and striving for status in a network context. As in the real social world, in this paper people do not live in isolated and closed communities. People have a variety of social contacts, and these contacts can overlap. People are indirectly connected, and therefore indirectly influenced, by people who may be quite distant in social and geographic space. We study how the status considerations and network structure influences individual outcomes and aggregate consumption of goods. Prominent scholars, such as Robert Frank (1985, 2000), argue that increasing inequality has led to excessive spending, as people try to emulate and compete with the rich. This process accelerates as more people are exposed to the lives of the rich and what they consume. We demonstrate these phenomena in a theoretical setting.

To study these phenomena we consider a game where individuals choose a consumption level for a given good. This good can be thought of as a private good with status implications, like cars or designer clothing. Alternatively, the good can be thought of as a public good – charity organizations, city opera houses, and the like – where individuals receive recognition in proportion to their contribution to the good in the form of published donor lists, for example. Agents gain value and incur individual costs as a function of their consumption level.

Agents additionally compare themselves in both relative and absolute terms to their neighbors in a network. We adopt a status loss model equivalent to that of [Stark and Wang 2005] where agents suffer a loss as they compare to themselves to those with higher consumption in their reference group. This utility specification finds its basis in the work of classic sociology and social psychology ([Merton 1938])and captures the idea that humans tend to look to those doing better than themselves to judge their relative position. Mathematically, the specification has features of both cardinal and ordinal social comparison. That is, an agent's loss due to social status is a function of both the degree to which his neighbors out-perform him and on the fraction of his

[1] See Truyts [2010] for a survey of the empirical work in this area

neighbors that are out-performing him. We feel that this is an important feature for a model of interpersonal-comparisons [2] For example, assume that agents compare themselves on the basis of brand quality of car. Imagine that an agent owns a Toyota Corolla (a cheaper car) and consider the following scenarios

(1) Most of the agent's neighbors also own a Corolla but one owns a Mustang (a medium-priced car).
(2) Most neighbors own a Corolla but one owns a Ferrari (an expensive car)
(3) All of the agent's neighbors own a Mustang.

In a model of purely ordinal social comparisons, the agent would experience the same status loss in scenarios 1 and 2 but this doesn't match with our intuitive notions of a status comparison. In a purely cardinal model, such as when an agent compares his consumption to the average consumption of neighbors, scenarios 2 and 3 are identical. In particular, the effect of an agent purchasing a Mustang would be the same in both scenarios. In our view, it seems reasonable that an agent would have a greater incentive to upgrade his car in scenario 3 because he needs to catch up to his entire neighborhood, rather than just a single high-consuming individual. We employ a simple model of social comparison that distinguishes these cases.

In our model, networks play a fundamental role as an agent's social comparison is only with his neighbors in the network. Much of the previous work in the social status literature addresses scenarios where each agent compares their consumption against the consumption patters of the entire population. While this approach is reasonable for a number of examples (for example, a small population of office workers comparing on suit quality), the network model is allows for a more subtle treatment of reference groups. For example, I may not choose to compare myself against the consumption patterns of celebrities but my friends might choose to do so. If their consumption pattern changes because of these celebrities and, in turn, cause my consumption to change, then I was indirectly influenced by the consumption of celebrities that I claimed to not care about. This phenomena, and others like it, raise a host of interesting research questions. How does the global structure of the network change the set of equilibria? How does an individual agent's actions vary as a function of his place in the network?

Our model is one of the first to address social comparisons in such a network setting. This paper tackles the primary theoretical challenges of this endeavor. Our primary objective is to solve for and analyze the Nash equilibria of the model.

We find that this model induces a supermodular game and thus exhibits strategic complementarities between the consumption levels of neighbors in the social network. Using tools from the supermodular game literature, we show that consumption at the minimum and maximum equilibria increases with status considerations, indicating that external organizations like charities or luxury good retailers have incentives to perpetuate the perception of their goods as status symbols. Welfare decreases with status concerns but individual utility decreases at varying rates, according to the degree of status spending surrounding each agent. This supports the intuition that attempts to "keep up with the Joneses," as people start to care more and more about the Joneses, cause agents to over-spend. This happens for all agents, even those that don't even know the Joneses, demonstrating the spillover effects of the network structure.

We conclude with a study of the effect of network structure on consumption and welfare. We characterize the set of supportable equilibria of a network by a notion of connectivity of groups of nodes, called cohesion; namely for agents to exert high consumption in equilibrium, they must be well-connected to other high-consumption agents, who must also be connected to other high-consumption agents. Using this charac-

[2]Much of the previous work in the social status literature uses a comparison model that is either ordinal or cardinal, but not both. Two popular examples are comparing your consumption to the average consumption of your reference group or caring about your position in the distribution of consumption for your reference group

terization, we are able to show that adding edges to the network can either increase or decrease consumption (and also welfare) as it alters the connectivity of groups of nodes.

The remainder of this paper is laid out as follows. Section 2 discusses related work in the economics and computer science literature. In section 4, we solve for the Nash equilibria of this game and we give comparative statics results based on these equilibria in 5. In section 6, we discuss the connection between the network structure and the set of supportable equilibria. We conclude with a discussion of the benefits and shortcomings of this model and give a detailed overview of future directions for this work.

2. RELATED WORK

While the study of status-seeking behavior is mostly new to computer science, there is a variety of previous work on social status in economics. By and large, this work considers agents who compare themselves to the same fixed reference group. Early work in this area, where a continuum of agents split their budgets between a visible and non-visible good, showed that status concerns lead to overspending on visible goods. [3] Furthermore, these outcomes are Pareto-dominated by, and produce the same social hierarch as, the outcome that would have occurred if all agents ignored their status concern. However, the ability for poorer agents to increase their status via the visible good promotes all members of society to spend more, to guard against a loss in status. Later work in the social status literature focused on the welfare effects of changing the exogenous distribution of wealth (budgets). A more equal distribution of wealth (one which has a greater density around the center of the support) demonstrates increased conspicuous consumption amongst the middle class but a drop in conspicuous consumption by the poor and rich. However, in the case where there is a discrete number of agents, it was demonstrated that social status need not effect outcomes if agent's budgets are sufficiently spread out (i.e. when it is simply too costly to compete with neighbors). For our purposes, the important commonality of the work is the assumption that all agents' status is jointly determined in one group - be it a closed social community or the whole society.

The most closely related paper to ours is Ghiglino and Goyal [2010] who consider spending on a visible and non-visible good on a fixed social network. However, they focus on a full market model where each agent is endowed with an initial amount of each good and prices are determined endogenously by this game. Interestingly, they show that an agent's consumption of the visible good is determined by his network centrality.

In terms of theory, the work most relevant to our model of competitive status is the study of *social context games* in [Ashlagi et al. 2008]. This setting specifies a graph G, an underlying game H, and some aggregation function, such as max/min/mean. The payoff for an agent i depends on both the payoffs from H and the aggregation of payoffs for i's neighbors in H. By contrast, our work assumes no distinction between the payoffs due to the game and the payoffs due to social comparisons. [Brandt et al. 2009] study the complexity of computing equilibria in the social context of ranking functions, where the underlying game produces some ordinal ranking of players and each player's utility weakly increases as he improves his position in the ranking.

More widely, our work contributes to the research on games played on a fixed network. The game in our paper has several interpretations, including provision of public goods and charitable contributions, and thus we share applications with other network literature.[4]

[3]For example, Frank (1985) considers a model where agents produce a positional and non-positional good. When agents care about their position within society, agents will under produce the non-positional goods. [Clark and Oswald 1998]Oded Stark and You Qiang Wang (2005) [Stark and Wang 2005] considers the case of a finite set of players in but in a setting where consumption levels are fixed and agents are free to choose the set of agents that they associate with and thus base their relative status on a fixed number of full connected agents. [Stark and Wang 2005]

[4][Bramoulle and Kranton 2007] introduced public goods game and study the characteristics of the Nash equilibrium. [Galeotti et al. 2009] study the effect of change in the degree of a node under the various assumptions of the positive or negative

3. THE MODEL

There are n agents $V = \{1, 2, ..., n\}$ working to procure a good (e.g., income, luxury goods like automobiles, or public goods like charity contributions). Each agent i chooses an consumption level $e_i \in [0, \infty]$, representing the degree to which he produces the good. Let $\mathbf{e}$ denote the consumption profile, and $\mathbf{e}_{-i}$ denote the consumption profile of all agents other than i. Agents are arranged in a network according to a connected, undirected graph $G = (V, E)$. An edge between agents i and j indicates that each agent's payoff is directly affected by the other's consumption. We will denote the neighbors of agent i by $N_i = \{j \in V | (i, j) \in E\}$. The payoffs consist of two components:

(1) **Economic Costs and Benefits.** We specify a simple form of costs and benefits that captures the basic tradeoffs between individual costs and benefits and possibly positive consumption externalities. We suppose the consumption of the good is proportional to the consumption level, with an agent-specific proportionality factor $\alpha_i > 0$, referred to as the agent's *type*. Thus, given consumption profile $\mathbf{e}$, the contribution to the payoff of agent i is $\alpha_i \cdot e_i$. As is standard in economics literature, we assume the cost of consumption is quadratic in consumption level contributing $-\frac{1}{2}e_i^2$ to the payoff of agent i. With this specification, the network costs and benefits remain simple enough so we can engage our main interest which is social status. All results from sections 4 and 5 extend to general convex costs in the obvious manner [5].

(2) **Social Status.** Status concerns of agents cause them to experience a disutility from being less productive than their neighbors. Given consumption profile $\mathbf{e}$, we posit a status loss function $S(e_i, \mathbf{e}_{-i}; G) = -\sum_{j \in N_i} \frac{1}{|N_i|+1} \max\{e_j - e_i, 0\}$, adopted from that of [Stark and Wang 2005]. The status loss function contributes $\beta \cdot S(e_i, \mathbf{e}_{-i}; G)$ to agent i's payoff, where $\beta > 0$ controls the extent of the status effect on agents' payoffs. Note this implies agent i has lower status when he produces a lower level of good relative to his neighbors, and this status loss is higher as the gap between the quantity of i's good and his neighbors' goods increases.

Thus, in summary, given consumption profile $\mathbf{e}$, social network $G = (V, E)$, and parameters β, an agent i of type α_i has a payoff of:

$$u_i(\mathbf{e}; \alpha_i, \beta, G) = \alpha_i e_i - \frac{1}{2}e_i^2 - \beta \sum_{j \in N_i} \frac{1}{|N_i| + 1} \max\{e_j - e_i, 0\}.$$

When it is clear from the context, we drop the parameters and graph specification from the utility function.

We define the *status game* as follows. Given the graph structure $G = (V, E)$, agents simultaneously choose consumption levels $e_i \in [0, \infty)$ and receive payoffs $u_i(\mathbf{e}; G)$. We are interested in the Nash equilibria of the game. An consumption profile $\mathbf{e}^*$ is Nash equilibrium vector if and only if for all agents i, $u_i(e_i^*, \mathbf{e}_{-i}^*; G) \geq u_i(e_i, \mathbf{e}_{-i}^*; G)$ for all e_i.

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externalities imposed by the actions of neighbors. [Ballester and Calvó-Armengol 2007] show a connection between equilibrium strategies and network centrality in a game with a simple model of network externalities. [Conitzer and Sandholm 2004] consider a model of charitable contributions where agents specify amounts that they are willing to contribute, contingent upon other agents contributing certain amounts. They introduce a bidding language for agents to express their preferences and provide optimal algorithms for determining clearing contribution levels for a restricted class of bids. [Ghosh and Mahdian 2008] give a similar model where an individuals value for contribution to a charity is equal to the sum of contribution levels of all of the agent's neighbors minus his own contribution. They give a mechanism which has the maximum aggregate contribution as an equilibrium under the condition that the graph is strongly connected.

[5] All proofs for these sections are done with the general convex case. It remains the focus of future work to extend the results in section 6 to a more general setting

4. EQUILIBRIUM ANALYSIS

In this section, we first characterize the best-response function and then we demonstrate that this game is a *supermodular game*. We use this property to show (1) the existence of a maximum and minimum Nash equilibrium and (2) the total consumption at these equilibrium is (weakly) increasing in parameters α_i and β. We also prove that iterated best-response converges to these two equilibrium *polynomially* quickly from particular initial configurations.

Our game supports a potentially infinite set of Nash equilibria $\{\mathbf{e}\}$. In this section, we argue that these equilibria form a complete lattice[6] with respect to the partial order $\succeq$ defined by $\mathbf{e} \succeq \mathbf{e}'$ if $e_i \geq e'_i$ for all i, and that the maximum element $\mathbf{e}^{\max}$ and minimum element $\mathbf{e}^{\min}$ of the lattice can be reached in polynomial time by a natural best-response dynamic from particular initial configurations.

4.1 Best-Response Functions

In order to prove that the set of Nash equilibria form a complete lattice, we will define a best-response function whose fixed points are Nash equilibria and argue that this is an isotone function on the lattice of consumption profiles defined by partial order $\succeq$.

The best-response of agent i to consumption profile $\mathbf{e}$ is $B_i(\mathbf{e}_{-i}) \equiv \arg\max_e u_i(e, \mathbf{e}_{-i}; G)$. In solving this maximization problem, agent i must weigh the marginal economic benefits less the costs of his consumption, which is simply $\alpha_i - e_i$, against the marginal effect of his action on his status loss. To see the effect of e_i on agent i' s status, consider the following reformulation of the status loss function $S(e_i, \mathbf{e}_{-i}; G)$:

$$\begin{aligned} S(e_i, \mathbf{e}_{-i}; G) &= -\sum_{j \in N_i} \frac{1}{|N_i|+1} \max\{e_j - e_i, 0\} \\ &= \frac{-1}{|N_i|+1} \sum_{j|e_j > e_i} e_j + \frac{1}{|N_i|+1} \sum_{j|e_j > e_i} e_i \\ &= \frac{-1}{|N_i|+1} \sum_{j|e_j > e_i} e_j + e_i \frac{1}{|N_i|+1} \sum_{j|e_j > e_i} 1 \end{aligned}$$

The second term is e_i times the proportion of agents in i's neighborhood that are playing a higher action. This number $\frac{1}{|N_i|+1} \sum_{j|e_j>e_i} 1$ is key to our analysis, as it indicates an agent's relative rank in his neighborhood. We will denote the proportion of higher-action players as $p(e_i, \mathbf{e}_{-i}; G) \equiv \frac{1}{|N_i|+1} \sum_{j \in N_i|e_j > e_i} 1$. Now the status loss term can be written as

$$S(e_i, e_{-i}; G) = \frac{-1}{|N_i|+1} \sum_{j \in N_i | e_j > e_i} e_j + e_i \cdot p(e_i, \mathbf{e}_{-i}; G)$$

where $p(e_i, \mathbf{e}_{-i}; G)$ is piece-wise linear. Figure 1 in Appendix A.1 provides an example of the change in agent i's status loss as agent i changes his action.

Except at points of discontinuity, small positive change in consumption level also corresponds to a reduction in status loss of $\beta \cdot p(e_i, \mathbf{e}_{-\mathbf{i}}; G)$. Intuitively, any best-response consumption level should be at a point where the marginal cost of consumption is equal to the marginal gain in value for the good plus the marginal reduction in status loss (when well-defined). We formalize this intuition in the following proposition. The proof is deferred to Appendix B.

[6] Recall a complete lattice is a partially ordered set $(L, \succeq)$ in which each non-empty subset A of L has a greatest lower and least upper bound.In particular, the lattice itself has a maximum and minimum element.

Proposition 4.1. *Fix the consumption profile* $\mathbf{e_{-i}}$ *of other agents and let* $B_i(\mathbf{e_{-i}}) = \arg\max_{e_i} u_i(e_i, \mathbf{e_{-i}})$ *be the best-response of agent* i *to* $\mathbf{e_{-i}}$. *Then*

$$B_i(\mathbf{e_{-i}}) = \arg\min_{e_i}\{e_i \mid e_i \geq \alpha_i + \beta \cdot p(e_i, \mathbf{e_{-i}}; G)\}.$$

Note B_i is technically a function of the parameters α_i, β and the network structure G. As before, when these are clear from the context, we omit them from the function.

From a graphical perspective, the best-response for agent i is the minimum consumption level e_i such that the line $y = x$ lies weakly above the graph of $\alpha_i + \beta \cdot p(e_i, e_{-i}; G)$. Figure 2 provides examples which demonstrate this intuition. Figure 2a in Appendix A.1 shows a case where there is an exact consumption level e_i where the marginal cost of consumption is equal to the marginal gain plus the marginal reduction in status loss, i.e., $e_i = \alpha_i + \beta \cdot p(e_i, \mathbf{e_{-i}}; G)$. In the case shown in Figure 2b, there's no exact consumption level which balances the marginal costs and benefits because $p(e_i, \mathbf{e_{-i}}; G)$ exhibits a discontinuity when agent i increases his consumption level. To get a feel for why this is the best-response, playing any consumption level less than the crossing point causes a large increase in loss of status compared with the small decrease in cost of consumption while playing anything greater than this point has a larger marginal cost than marginal gain.

Corollary 4.2. *For any equilibrium action level* e_i^* *of agent* i, $e_i^* \in [\alpha_i, \alpha_i + \frac{|N_i|}{|N_i+1|}\beta]$.

We now show this game is a supermodular game, as defined by Milgrom and Roberts [1990]. Supermodular games are characterized by games that exhibit strategic complementarities and, in turn, have nice comparative statics properties. We make use of these properties in sections 4 and 5. A game is a supermodular game if it satisfies the following properties:

—Each player's strategy space is a compact subset of $\mathcal{R}$.
—u_i is upper semi-continuous in e_i and e_{-i}
—u_i has increasing differences in (e_i, e_{-i}); for $e'_i > e_i$ and $e'_{-i} > e_i$,

$$u_i(e'_i, e'_{-i}) - u_i(e_i, e'_{-i}) \geq u_i(e'_i, e'_{-i}) - u_i(e_i, e'_{-i})$$

Theorem 4.3. *The social status game is a supermodular game.*

Proof. The first property follows from the previous corollary while the second property is straightforward from the utility function. It remains to show why the utility function satisfies the increasing difference property. Consider two consumption levels for player i $e'_i > e_i$ and two consumption vectors for all other players $e'_{-i} \geq e_i$. First we would like to show

$$S(e'_i, e'_{-i}) - S(e_i, e'_{-i}) > S(e_i, e'_{-i}) - S(e_i, e_{-i})$$

Each agent $j \in N_i$ contributes a status loss of $\frac{\max\{e_j - e_i, 0\}}{|N_i|+1}$ and thus raising consumption from e_i to e'_i reduces the status loss that agent j imposes on i by $\frac{\min\{e_j - e_i, e'_i - e_i\}}{|N_i|+1}$. If we raise e_j to some e'_j, then the change in agent i's consumption level becomes (weakly) more valuable. Since the status function satisfies the increasing difference property and value and cost of consumption do not depend on other agents, $u_i(e_i, \mathbf{e_{-i}})$ has increasing differences in $(e_i, \mathbf{e_{-i}})$. □

Theorem 4.4. *Amongst the non-empty set of all Nash equilibria, there exists a maximum equilibrium* $\mathbf{e}^{\max}$ *and minimum equilibrium* $\mathbf{e}^{\min}$.

That is, the consumption level of agent i in $\mathbf{e}^{\max}$ ($\mathbf{e}^{\min}$) is greater (less) than his consumption in any other equilibrium

4.2 Best-Responses Dynamics

In this section we consider best-response dynamics in our setting. While the convergence property may follow from the supermodular property, we go further and show these dynamics converge in polynomial time. In our setting, this corresponds to a best-response dynamic in which agents update strategies simultaneously in each round. Arguably more natural is a sequential best-response dynamic in which, in each round each agent i sequentially computes a best-response to the consumptions of other agents. We first formally define the sequential best-response dynamics. Label agents in an arbitrary order $\{1, \dots, n\}$. We allow agents to update consumptions in round-robin fashion according to that ordering as shown in the pseudocode of Algorithm 1.[7]

Algorithm 1 Best-Response($\mathbf{e}^0$)

```
t ← 0
repeat
  e ← e^t
  for i = 1 to n do
    e_i ← arg max_e u_i(e, e_{-i}; G)
  end for
  e^{t+1} ← e
  t ← t + 1
until e^t = e^{t-1}
```

Here we prove via a standard potential function argument that these dynamics also converge and moreover do so in polynomial time.

THEOREM 4.5. *For an consumption level profile $\mathbf{e}^0$ where $e_i^0 = \alpha_i + \frac{|N_i|}{|N_i|+1}\beta$, Algorithm 1 on input $\mathbf{e}^0$ converges in time $O(n^4)$ to the maximum Nash equilibrium $\mathbf{e}^{\max}$. Similarly, for an consumption level profile $\mathbf{e}^0$ where $e_i^0 = \alpha_i$, Algorithm 1 on input $\mathbf{e}^0$ converges in time $O(n^4)$ to the minimum Nash equilibrium $\mathbf{e}^{\min}$.*

PROOF. We prove the first claim, that Algorithm 1 on input $e_i^0 = \alpha_i + \frac{|N_i|}{|N_i|+1}\beta$ converges to $\mathbf{e}^{\max}$ in polynomial time (the proof of the second claim is similar). We first show by induction that at every round $t \geq 1$ until the last round of Algorithm 1, the consumption profile $\mathbf{e}^t$ is strictly dominated by $\mathbf{e}^{t-1}$ according to partial order $\succeq$ and so the consumption profiles computed by the algorithm form a chain in the lattice. Note that in $\mathbf{e}^0$, each agent exerts an consumption at least as large as his consumption in any equilibrium by Corollary 4.2. Thus after the first round, each agent's consumption weakly decreases and at least one agent's consumption strictly decreases (otherwise we have reached an equilibrium). Now consider an arbitrary round t. In this round, by induction each agent best-replies to an intermediate consumption profile that is weakly dominated by the one he considered in his last best-response computation. Hence, by Proposition 4.1, each agent's consumption weakly decreases and at least one agent's consumption strictly decreases (otherwise we have reached an equilibrium).

We conclude by noting that due to the form of the best-response function as stated in Proposition 4.1, each agent's consumption in any best-response is either a constant plus one of $|N_i|$ multiples of $p(e_i, \mathbf{e}^{-i}; G) = \frac{1}{|N_i|+1} \sum_{j \in N_i | e_j > e_i} 1$, in case where there exists an $B(\mathbf{e_{-i}}) = \alpha_i + \beta p(B(\mathbf{e_{-i}}), \mathbf{e_{-i}})$, or the consumption level of a neighbor, i.e. $e_i^* = e_j$ for some j, in the case where $B(\mathbf{e_{-i}}) > \alpha_i + \beta p(B(\mathbf{e_{-i}}), \mathbf{e_{-i}})$. By induction, any

[7]Actually, the order of updates is not important; as can be seen from the following proof, so long as each agent updates at least every d steps, the dynamic converges in time polynomial in d and n.

neighbor consumption must be in the from $e_j = \alpha_j + \beta \frac{m}{|N_k|+1}$ for some m and $k \in N_j$. Thus each best-response function B_i takes on at most n^2 distinct values (he can copy up to n agents, who each have n distinct values they can contribute). Therefore, the maximum length of a chain in the lattice is n^3. Therefore the algorithm can take at most n^3 rounds, and so converges in time $O(n^4)$ as claimed. $\square$

5. ECONOMIC RESULTS

In this section, we explore various properties of equilibrium consumption levels, including the amount of consumption of the good and the social welfare of the agents. We are particularly interested in the effects of status considerations on these properties. In absence of status effects (i.e., $\beta = 0$), an agent's utility-maximizing consumption level is equal to his type, $e_i = \alpha_i$, regardless of the consumptions of others $\mathbf{e}_{-i}$. This is because at consumption level $e_i = \alpha_i$, the marginal contribution of consumption $-e_i + \alpha_i$ is zero (when $\beta = 0$). Thus these consumption levels define the unique Nash equilibrium when $\beta = 0$ and will be our baseline for comparison.

Definition 5.1. The *status-free consumption profile* $\mathbf{e}^{sf}$ is defined by $e_i^{sf} = \alpha_i$.

We will compare the properties of this status-free consumption profile to the properties of the minimum equilibrium $\mathbf{e}^{\min}$ and the maximum one $\mathbf{e}^{\max}$ for various settings of the parameters and network structures.

5.1 Consumption

We first study the effects of the parameters on consumption in equilibrium. We define the *consumption* of the good at profile $\mathbf{e}$ to be $\sum_{i=1}^{n} e_i$. The level of consumption is of particular interest when the good has some public benefit, e.g., contribution to a charity. Naturally consumption increases with type and status concerns.

THEOREM 5.2. *The minimum-producing equilibrium is* $\mathbf{e}^{\min}$ *and the maximum-producing one is* $\mathbf{e}^{\max}$. *Furthermore,*

—*consumption at* $\mathbf{e}^{\min}$ *is increasing in* α_i *and weakly increasing with respect to* β,
—*and consumption at* $\mathbf{e}^{\max}$ *is increasing in both* α_i *and* β.

The above theorem implies that consumption in equilibria weakly dominates consumption in the status-free consumption profile. In fact, it is not hard to see that in heterogenous settings, this comparison is strict: the consumption of the status-free consumption profile is strictly less than any equilibrium.

5.2 Welfare

We next study the effects of the parameters on welfare in equilibrium. We define the *social welfare* of profile $\mathbf{e}$ to be $\sum_{i=1}^{n} u_i(\mathbf{e}; G)$. Naturally, in this case, the status-free consumption profile maximizes welfare, the minimum-equilbrium $\mathbf{e}^{\min}$ is the maximum-welfare equilibrium, and the maximum-equilibrium $\mathbf{e}^{\max}$ is the minimum-welfare one. This is because profiles with higher consumption can only decrease each agent's payoff as $u_i(e_i, \mathbf{e}_{-i}; G)$ is maximized at $e_i = \alpha_i$ and decreasing in e_j for $j \neq i$.

We show that raising β causes a direct loss of welfare stemming from a higher cost of equilibrium consumption and a greater weight on loss due to social status. However, raising β can cause an indirect loss or gain in social status depending on the magnitude of an agent's status premium in comparison to their neighbors.

We first show that holding other agents' consumption fixed, the equilibrium utility for an agent is decreasing in β, the agent's concern for status.

PROPOSITION 5.3. *Consider a fixed setting of parameters* α_i, β *and* $\delta = 0$, *and fix any consumption profile* $\mathbf{e}$ *for these parameters. Let* $\beta' > \beta$, *choose an arbitrary agent* i, *and let* $e_i = B(\mathbf{e}_{-i}; \alpha_i, \beta, G)$ *and*

$e_i' = B(\mathbf{e}_{-i}; \alpha_i, \beta', G)$ be agent i's best-response to the old and new parameters, respectively. Then $e_i' > e_i$ and $u_i(e_i, \mathbf{e}_{-i}; \alpha_i, \beta, G) \geq u_i(e_{-i}', \mathbf{e}_{-i}; \alpha_i, \beta', G)$.

While increasing agent i's concern for status might cause him to raise his consumption and actually reduce his status loss, the increased cost of consumption and increased weight on status loss outweighs any value gained.

We next show that increasing status concern for all agents results in (weakly) lower welfares for all agents in both $\mathbf{e}^{\max}$ and $\mathbf{e}^{\min}$.

PROPOSITION 5.4. *Consider a fixed setting of parameters α_i, β. Let $\beta' > \beta$. Then for all i, the following two inequalities hold*

$$u_i(\mathbf{e}^{\max}; \alpha_i, \beta, G) \geq u_i(\mathbf{e}^{\max}; \alpha_i, \beta', G)$$

$$u_i(\mathbf{e}^{\min}; \alpha_i, \beta, G) \geq u_i(\mathbf{e}^{\min}; \alpha_i, \beta', G)$$

To be clear, $\mathbf{e}^{\max}$ and $\mathbf{e}^{\min}$ refer to the maximum and minimum equilibrium for the particular set of parameters.

Raising β causes a welfare loss in two ways: it raises each agent's consumption, which in turn raises their cost of consumption, and increases the concern each agent has for status, which only causes a loss to each agent. Additionally, there is the indirect effect that the raised consumption by an agent's neighbors might further increase the difference in consumption between the agent and his neighbors, further increasing his loss from social status. Recall that the equilibrium consumption for agent i is the minimum e_i such that $e_i \geq \alpha_i + \beta p(e_i, \mathbf{e}_{-\mathbf{i}})$. For all agents i, define f_i such that $e_i = \alpha_i + f_i\beta$. We refer to the term $f_i\beta$ as an agent's *status premium*. We use status premium to distinguish between neighbors of agent i who have a higher consumption: those who consume $e_j > e_i$ because they have a greater value for the good and those who consume more because they have a higher status premium. In the next proposition, we show that

PROPOSITION 5.5. *Fix a set of agents and a graph G and a setting where $|\alpha_i - \alpha_j| > \beta$ for all i, j. Let $\mathbf{e}^{\max}$ denote the maximum equilibrium for this game. For each agent i, the marginal status loss at the maximum equilibrium with respect to β is equal to*

$$\frac{1}{|N_i| + 1} \sum_{\mathbf{e}_{\mathbf{j}}^{\max} > \mathbf{e}_{\mathbf{i}}^{\max}} ((\alpha_j - \alpha_i) + 2\beta(f_j - f_i))$$

This holds analogously for $e^{\min}$.

The above equation demonstrates the direct and indirect effect on status loss with respect to β. There is a constant marginal status loss of $\alpha_j - \alpha_i$ caused by raising concern for status. The role of the status premium in the above equation reflects the additional status loss due to increased consumption. Neighbors with a higher status premium induce an increasing marginal status loss to agent i while neighbors with a lower status premium than i actually have a decreasing marginal status loss on agent i. Note that the assumption that $|\alpha_j - \alpha_i| > \beta$ is to ensure that the equilibrium rank of an agent in his neighborhood does not change as β changes. If this condition fails, it could be that raising β actually decreases the total status loss than an agent experiences, however he would still suffer a loss of welfare overall due to his increased cost of consumption.

6. NETWORK EFFECTS

We conclude by studying the effect of the social network structure on both consumption and welfare. In particular, we discuss the implications of adding or deleting links from the network. For clarity, we focus on the setting where agents have a homogenous type $\alpha_i = \alpha$ (in which case $\mathbf{e}^{sf} = \mathbf{e}^{\min}$). In such settings, we can derive the following general characterization of consumption levels sustainable in equilibrium in terms of the *cohesion* of nodes in the network.

Definition 6.1. For a given set of nodes T, we define the *cohesion* of T to be the largest number ρ such that for any node $i \in T$, at least a ρ fraction of N_i are also in T. We say a set of nodes T is ρ-*cohesive* if the cohesion of T is at least ρ. Formally, T is ρ-cohesive if $\forall i \in T$, $\frac{|N_i \cap T|}{|N_i|+1} \geq \rho$.

THEOREM 6.2. *Given consumption profile* $\mathbf{e}$*, define* f_i *such that* $e_i = \alpha + f_i\beta$ *for all* i*. Let* $S_f = \{i \in N | f_i \geq f\}$*. If* $\mathbf{e}$ *is an equilibrium, then* S_f *is* f*-cohesive for all* $f \in [0, \frac{|N|}{|N|+1}]$.

Intuitively, if an agent i is playing $\mathbf{e}_i = \alpha_i + f_i\beta$ and S_{f_i} is not f_i-cohesive, then he could reduce his consumption by a small amount without affecting his rank is his neighborhood. This would imply his original consumption was too high and thus $\mathbf{e}$ would not have been an equilibrium. The full proof is deferred to the appendix.

From the above theorem, it is clear that adding or deleting links will change the set of supportable equilibria as it changes the cohesion of subsets of nodes. Indeed, the following examples show that these effects can have arbitrary consequences for consumption and welfare.

Example 6.3. Consider the homogenous setting where $\alpha_i = \alpha$ and agents arranged according to the complete graph on 10 nodes, K_{10}, and a complete graph on 3 nodes K_3 such that there's an edge between node $i \in K_{10}$ and $j \in K_3$. This graph is shown in 3a. The maximum consumption level of node j is $\alpha + \frac{3}{4}\beta$. Thus for any $e > \alpha + \frac{3}{4}\beta$, at most a $\frac{9}{11}$ fraction of i's neighborhood can play e in equilibrium (the i's 9 neighbors in K_{10} can play a high consumption level but j cannot). Then, by theorem 6.2, i can play at most $\alpha + \frac{9}{11}\beta$ in equilibrium. Indeed, $\mathbf{e}^{\max}$ on this graph is $e = \alpha + \frac{9}{11}\beta$ for all the nodes in K_{10} and $e = \alpha + \frac{2}{3}\beta$ for all nodes in K_3. However, if we add the edge between agent i and another node $k \in K_3$, then for any $e > \alpha + \frac{3}{4}\beta$, the cohesion in i's neighborhood drops to $\frac{9}{12}$, implying that agent i can play at most $\alpha + \frac{9}{12}\beta$ in $\mathbf{e}^{\max}$. Additionally, this edge didn't raise the $\mathbf{e}^{\max}$ consumption levels for the nodes in K_3 because the cohesion is bounded at $\frac{2}{3}$ due to the node in K_3 that doesn't have an edge with i. Thus adding this edge strictly decreased the consumption of the good.

7. CONCLUSION

This paper presents a network of social status. Agents have economic costs and benefits of providing a good and they compare their production to those in their neighborhood. When a graph is connected but not complete, agents' incentives are affected by not only by their neighbors, but by actions of agents at a distance in the network. We study the Nash equilibria of the game; we find the equilibrium set forms a lattice. Best response dynamics naturally converge to either the minimum or maximum equilibria. We compare equilibrium outcomes for different types of agents and show that striving for status has significant impact on agents' production and welfare. While status concerns increase aggregate production, they decrease social welfare (even pointwise) as agents over-produce the good relative to the economic costs and benefits. Furthermore, adding or deleting links can affect production (and also welfare) in arbitrary ways.

We leave open for future work the obvious and interesting network design questions. Given a fixed network and initial agent types α_i, how should a designer with limited resources increase types to maximize production? Such a strategy amounts to marketing the good to particular agents in the network. Alternatively, Given fixed types α_i and an initial network, how should a designer with limited resources introduce edges to maximize production? This strategy amounts to introducing particular agents in the network, e.g., through charity balls.

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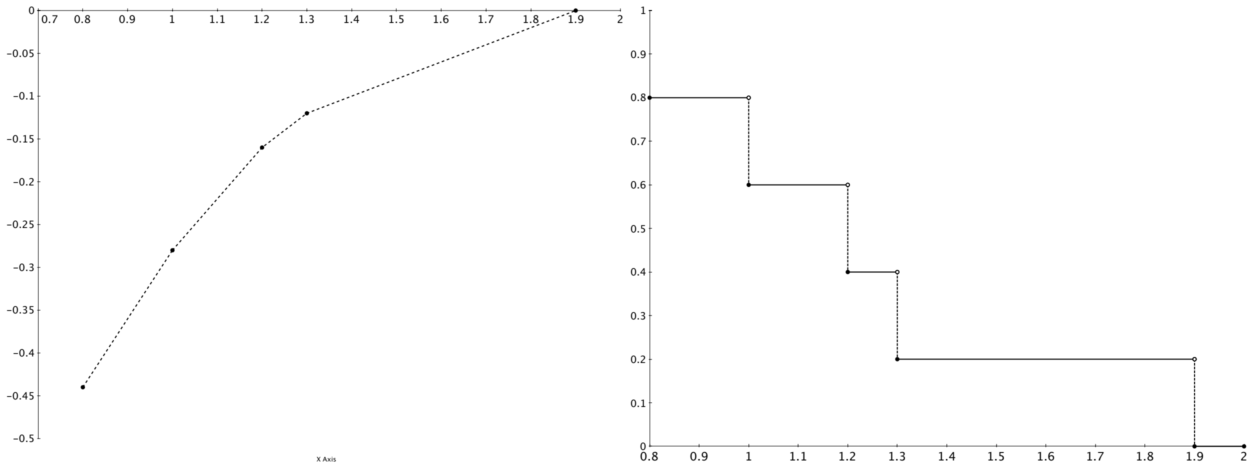
A. APPENDIX

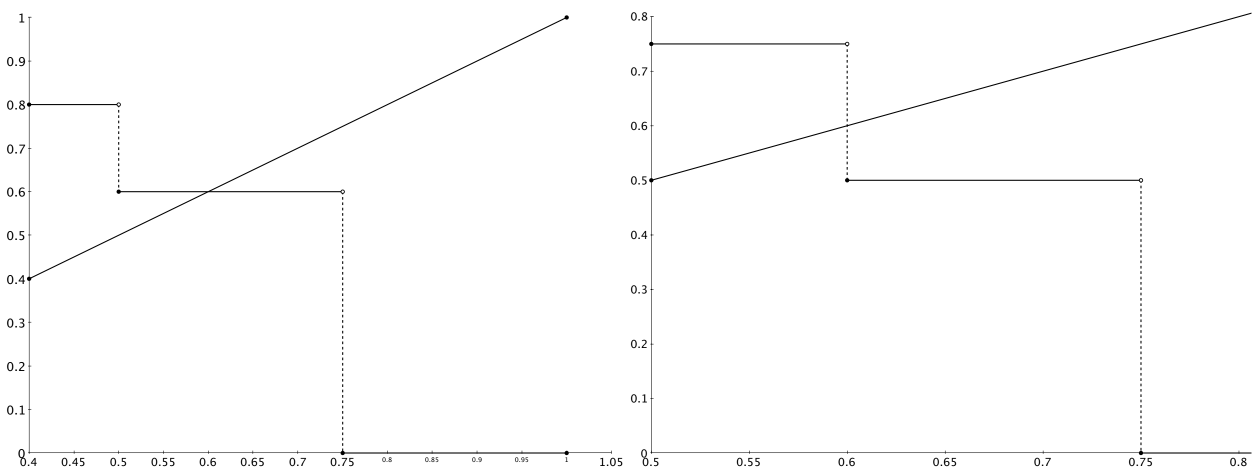
A.1 Missing Figures



(a) Status Loss Curve

(b) Marginal Status Loss Curve

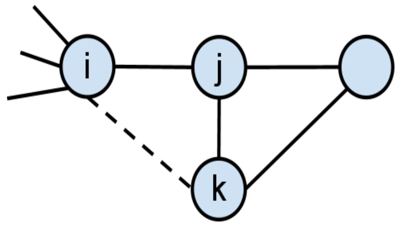
Fig. 1: Figure 1a plots the status loss curve as function of consumption of an agent whose neighbors' consumption levels are $(1, 1.2, 1.3, 1.9)$. Figure 1b plots the marginal loss in status as a function of consumption when faced with the same profile.



(a) Best-Response Curves

(b) Best-Response Curve

Fig. 2: Given fixed consumption levels $\mathbf{e}_{-i}$, we plot the curve $\alpha_i + \beta \cdot p(e_i; \mathbf{e}_{-i}; G)$ as a function of consumption level e_i. The intersection point between this curve and the line $y = x$ is the best-response of agent i. Figure 2a (respectively 2b) depicts the curve when the consumption levels of i's neighbors are $(0.75, 0.75, 0.75, 0.5)$ (respectively $(0.6, 0.75, 0.75)$).



(a) Homogenous Case

Fig. 3: Figure 3a shows a graph for which adding an edge decreases $\mathbf{e}^{\max}$, as in example 6.3. The entire K_{10}, with the exception of node i, is not shown. In this example, adding the edge represented by the dotted line decreases the consumption level that agent i plays in $\mathbf{e}^{\max}$ and in fact causes aggregate consumption to decrease on this graph.

B. MISSING PROOFS FROM SECTION 4

B.1 Useful lemma

To prove Proposition 4.1, we first establish a lemma about the reduction in status loss when an agent raises his consumption from e_i to $e_i + x$ (holding fixed all other agents' consumptions).

LEMMA B.1. *For any e_i and $x > 0$, the difference in social status effects $S(e_i + x; \mathbf{e_{-i}}; G) - S(e_i, \mathbf{e_{-i}}; G)$ is equal to the following*

$$S(e + x, \mathbf{e_{-i}}) - S(e, \mathbf{e_{-i}}) = \int_{e}^{e+x} p(y, \mathbf{e_{-i}})dy$$

PROOF. Reorder e_{-i} such that $e_1 \leq e_2$..... As previously shown, $\frac{\delta S(e_i, e_{-i})}{\delta e_i} = p(e_i, e_{-i})$ for any $e_i \in [e_j, e_{j+1})$. We can also say that $\frac{\delta S(e_i, e_{-i})}{\delta e_i} = p(e_i, e_{-i})$ for any $e_i \in [e_j, e_{j+1} - \epsilon]$ for small $\epsilon > 0$.

WLOG, assume that $e_{i-1} \leq e_i \leq e_{i+1} ... \leq e_{i+k} \leq e_i + x$. That is, let $e_{i+1}, ...e_{i+k}$ be the set of neighbor consumption levels that are greater than e_i but less than $e_i + x$. For convenience, let $e_{k+1} = e_i + x$. Then we can write $S(e_i + x; \mathbf{e_{-i}}) - S(e_i, \mathbf{e_{-i}})$ as the following telescoping sum

$$S(e_i + x; \mathbf{e_{-i}}) - S(e_i, \mathbf{e_{-i}}) = \sum_{j=i}^{k} S(e_{j+1}, e_{-i}) - S(e_j, e_{-i})$$

Then for any interval, $[e_j, e_{j+1} - \epsilon]$, $S(e_{j+1} - \epsilon, e_{-i}) - S(e_j, e_{-i}) = \int_{e_j}^{e_{j+1}-\epsilon} p(y, e_{-i})\delta y$. (Since $p(e_i, e_{-i})$ is monotonic, it is integrable). This yields

$$S(e_i + x; \mathbf{e_{-i}}) - S(e_i, \mathbf{e_{-i}}) = \sum_{j=i}^{k+1} \int_{e_j}^{e_{j+1}-\epsilon} p(y, e_{-i})\delta y$$

Letting $\epsilon \to 0$ and using linearity of integration gives the result

$$S(e+x, \mathbf{e_{-i}}) - S(e, \mathbf{e_{-i}}) = \int_e^{e+x} p(y, \mathbf{e_{-i}})dy$$

□

B.2 Proof of 4.1

We will now prove theorem 4.1 but we will do so using a more general utility function. Specifically, we assume convex costs rather than quadratic. 4.1 then follows as a corollary.

PROPOSITION B.2. *Let $u_i(e_i, e_{-i}) = \alpha_i + \beta S(e_i, e_{-i}) - C(e_i)$ where $C(e_i)$ is a twice-differentiable convex function representing the cost of consumption. Denote the first derivative of $C(e_i)$ by $c(e_i)$. Fix the consumption profile $\mathbf{e_{-i}}$ of other agents and let $B_i(\mathbf{e_{-i}}) = \arg\max_{e_i} u_i(e_i, \mathbf{e_{-i}})$ be the best-response of agent i to $\mathbf{e_{-i}}$. Then*

$$B_i(\mathbf{e_{-i}}) = \arg\min_{e_i}\{e_i \;|c^{-1}(e_i \geq \alpha_i + \beta \cdot p(e_i, \mathbf{e_{-i}}; G))\}.$$

PROOF. Define the set $Y = \{e_i | c(e_i) \geq \alpha_i + \beta p(e_i, \mathbf{e_{-i}}; G)\}$ and let $e_i^* = \min_{e_i \in Y} e_i$. This proof is broken down into two parts; First, we show that e_i^* exists, then show that $u_i(e_i^*, \mathbf{e_{-i}}; G) > u_i(e_i', \mathbf{e_{-i}}; G)$ for all consumption levels $e_i' \neq e_i^*$.

First we must establish the existence of a minimum element of Y. Fix the vector $\mathbf{e_{-i}}$ and reorder such that $e_1 \leq e_2$.... Then for any $e_j \neq e_{j+1}$, any $e_i \in [e_j, e_{j+1})$ satisfies $p(e_i, \mathbf{e_{-i}}; G) = p(e_j, \mathbf{e_{-i}}; G)$ because $p(e_i, \mathbf{e_{-i}}; G)$ is defined as the proportion of i's neighbors that are playing a strictly higher consumption level than e_i.

We now use this claim to show that e_i^* is the unique maximizer of $u_i(\cdot, \mathbf{e_{-i}}; G)$.

—e^* is better than any lower consumption level. For any $x > 0$, consider $u_i(e_i^*) - u_i(e_i^* - x) =$

$$\alpha_i \cdot (e_i^*) - S(e_i^*, \mathbf{e_{-i}}; G) - C(e_i^*) - \Big(\alpha_i \cdot (e_i^* - x) - S(e_i^* - x, \mathbf{e_{-i}}; G) - C(e_i^* - x)\Big)$$

$$= \alpha_i \cdot x + \beta(S(e_i^*) - S(e_i^* - x)) - \int_{e_i^*-x}^{e_i^*} c(y)dy$$

By definition, all $e_i' < e_i^*$ satisfy

$$c(e_i') < \alpha_i + \beta p(e_i', \mathbf{e_{-i}})$$

Yielding

$$\alpha_i \cdot x + \beta(S(e_i^*) - S(e_i^* - x)) - \int_{e_i^*-x}^{e_i^*} c(y)dy > \alpha_i \cdot x + \beta(S(e_i^*) - S(e_i^* - x)) - \int_{e_i^*-x}^{e_i^*} \alpha_i + \beta p(y, \mathbf{e_{-i}})dy$$

$$= \beta(S(e_i^*) - S(e_i^* - x)) - \beta \int_{e_i^*-x}^{e} p(y, \mathbf{e_{-i}})dy$$

By lemma B.1 to the $S(e_i^*) - S(e_i^* - x)$ term , we get

$$= \beta \int_{e_i^*-x}^{e} p(y, \mathbf{e_{-i}})dy - \beta \int_{e_i^*-x}^{e} p(y, \mathbf{e_{-i}})dy = 0$$

Tying it all together gives $u_i(e_i^*) - u_i(e_i^* - x) > 0$

—e^* is better than any higher consumption level. For any $x > 0$, consider $u_i(e_i^*) - u_i(e_i^* + x) =$

$$\alpha_i \cdot (e_i^*) - S(e_i^*, \mathbf{e_{-i}}; G) - C(e_i^*) - \Big(\alpha_i \cdot (e_i^* + x) - S(e_i^* + x, \mathbf{e_{-i}}; G) - C(e_i^* + x)\Big)$$

$$= -\alpha_i \cdot x - \beta(S(e_i^* + x) - S(e_i^*)) + \int_{e_i^*}^{e_i^*+x} c(y)dy$$

By definition, any $e_i' > e_i^*$ satisfies $c(e') > \alpha_i + \beta p(e_i', \mathbf{e_{-i}})$. Combining this with lemma B.1 yields

$$> -\alpha_i \cdot x - \beta \int_{e_i^*}^{e_i^*+x} p(y, \mathbf{e_{-i}})dy + x\alpha_i + \beta \int_{e_i^*}^{e_i^*+x} p(y, \mathbf{e_{-i}})dy = 0$$

Tying it all together shows $u_i(e_i^*) - u_i(e_i^* + x) > 0$

Thus playing e_i^* is the unique best-response for player i to consumption vector $\mathbf{e_{-i}}$. □

C. MISSING PROOFS FROM SECTION 5

C.1 Proof of theorem 5.2

We show the comparative statics on consumption via a corollary of supermodular game. Milgrom and Roberts [1990] show the following

THEOREM C.1. *Suppose that the payoff function in a supermodular game are parameterized by t such that $u()$ has increasing differences in (e_i, t). Then the maximum and minimum equilibrium are non-decreasing in t.*

PROPOSITION C.2. *$u_i(e_i, \mathbf{e_{-i}}, \alpha_i, \beta)$ has increasing differences in (e_i, α_i) and (e_i, β).*

PROOF. Consider $e_i' > e_i$, $\alpha_i' > \alpha_i$ and $\beta' > \beta$.

$$(u_i(e_i', \mathbf{e_{-i}}, \alpha_i', \beta) - u_i(e_i, \mathbf{e_{-i}}, \alpha_i', \beta)) - (u_i(e_i', \mathbf{e_{-i}}, \alpha_i, \beta) - u_i(e_i, \mathbf{e_{-i}}, \alpha_i, \beta))$$

$$= (\alpha_i' - \alpha_i)(e_i' - e_i) > 0$$

So u_i satisfies ID in (e_i, α_i).

$$(u_i(e_i', \mathbf{e_{-i}}, \alpha_i, \beta') - u_i(e_i, \mathbf{e_{-i}}, \alpha_i, \beta')) - (u_i(e_i', \mathbf{e_{-i}}, \alpha_i, \beta) - u_i(e_i, \mathbf{e_{-i}}, \alpha_i, \beta))$$

$$= (\beta' - \beta)(S(e_i', \mathbf{e_{-i}}) - S(e_i, \mathbf{e_{-i}})) > 0$$

So u_i satisfies ID in (e_i, β). □

Theorem 5.2 now follows from C.2 and the above property of supermodular games.

C.2 Proof of theorem 5.3

We now prove theorem 5.3 in the more general convex setting.

PROOF. Note by the form of the best-response function given in Proposition 4.1, B_i is weakly increasing in β and so $e_i' \geq e_i$. Let $\sigma = e_i' - e_i$, let $\epsilon = \beta' - \beta$, and note both σ and ϵ are non-negative. Consider the utility of agent i in the two settings:

$$u_i(e_i; \alpha_i, \beta, G) = \alpha_i e_i + \beta S(e_i) - C(e_i)$$

and

$$u_i(e_i'; \alpha_i, \beta', G) = \alpha_i(e_i + \sigma) + (\beta + \epsilon)S(e_i') - C(e_i + \sigma)$$

Subtract the first from the second

$$\alpha_i\sigma + \epsilon S(e_i + \sigma) + \beta(S(e_i + \sigma) - S(e_i)) - (C(e_i + \sigma) - C(e_i))$$

As we saw in the proof of 4.1, lemma B.1 implies the following

$$\alpha_i\sigma + \beta(S(e_i + \sigma) - S(e_i)) - (C(e_i + \sigma) - C(e_i)) < 0$$

By definition, $\epsilon S(e_i + \sigma) \leq 0$. Hence the welfare of agent i weakly decreased, and strictly decreased if any of the terms (e.g., the status) is negative.

□

D. MISSING PROOFS FROM SECTION 6

D.1 Proof of theorem 6.2

PROOF. Pick an arbitrary $f \in [0, \frac{|N}{|N|+1}]$ and let agent i be a member of S_f. For all agents j, define f_j such that $\mathbf{e_j} = \alpha + f_j\beta$. Note that in this setting, $\mathbf{e_i} > \mathbf{e_j}$ iff $f_i > f_j$. Let $g(f, \mathbf{e_{-i}}; G) = \frac{|N_i \cap S_f|}{|N_i+1|}$ be the fraction of i's neighborhood that is playing $f_j \geq f$ in $\mathbf{e_{-i}}$. For the sake of contradiction, assume that agent i's neighborhood is not $f-$cohesive, i.e.

$$f > g(f, \mathbf{e_{-i}}; G)$$

Then we will show that there exists a small negative deviation for agent i which satisfies the conditions of theorem 4.1, contradicting that $\mathbf{e_i}$ is the best-response for player i (and that $\mathbf{e}$ is an equilibrium).
Then let $\sigma = f - g(f, \mathbf{e_{-i}}; G)$, let $\epsilon = \min_{j \in N_i | f_j < f} f - f_j$, and let $\theta = \min\{\sigma, \epsilon\}$. Because θ is small, any agent j such that $f_j < f$ also satisfies $f_j < f - \frac{\theta}{2}$. This implies

$$g(f, \mathbf{e_{-i}}; G) = g(f - \frac{\theta}{2}, \mathbf{e_{-i}}; G)$$

This yields the following

$$f - \frac{\theta}{2} > g(f - \frac{\theta}{2}, \mathbf{e_{-i}}; G) \geq p(f - \frac{\theta}{2}, \mathbf{e_{-i}}; G)$$

Where the last inequality holds because $g()$ gives the fraction i's neighbors that are playing a weakly higher consumption level while $p()$ gives the fraction of i's neighbors that are playing a strictly higher consumption level. Finally the above inequality gives, $\alpha + (f - \frac{\theta}{2})\beta \geq \alpha + p(f - \frac{\theta}{2}, \mathbf{e_{-i}}; G)\beta$, so it satisfies the conditions in theorem 4.1 for a best-response. This contradicts that e_i was the best-response for player i.

So our assumption that player i's neighborhood was not f-cohesive is false. Then for any agent in S_f, their neighborhood must be at least f-cohesive in S_f. Thus S_f is f-cohesive. □