Northwestern University
Department of Electrical and Computer Engineering

ECE 428: Information Theory — Spring 2004

Problem Set 8 — **Date issued: May 25, 2004**
Date Due: June 3, 2004

Reading Assignment: Chapter 10

Do the following problems:

1. Problem 10.1 in C&T. Justify the steps leading to (10.142), (10.143), and (10.144). Justify (10.143) in part by showing the more general result, that for any function $f(a,b)$,

$$\min_a \max_b f(a,b) \geq \max_b \min_a f(a,b).$$

2. Problem 10.3 in C&T.

3. Problem 10.4 in C&T.

4. **Enough Bandwidth:** Let C_∞ denote the capacity of an infinite bandwidth power-limited additive white Gaussian noise channel. Find the bandwidth W_{95} such that the bandwidth constraint reduces the capacity to $0.95C_\infty$. The *spectral efficiency* of a bandlimited channel is the rate in bits per second per hertz. Numerically, find the spectral efficiency of a system using the above bandwidth?

Discuss whether high or low spectral efficiency is reasonable from the perspective of a communication system designer.

5. Consider a set of N independent, discrete-time white Gaussian noise channels that can be used in parallel. The noise variance for the nth channel is given by $\sigma_n^2 = n^2$ for each n. The input signal is constrained by the condition $\sum_{i=1}^{N} P_i / n \leq 6$, where P_i represents the power used on the nth channel.

a. For N=2, what is the capacity of the set of parallel channels. (*Hint: if you set this up correctly, you should not have to do any complicated calculations.*)

b. Redo part (a) for $N = \infty$.