## Binary trees

; a database is
; - false
; - (make-db-node person database database)
(define-struct db-node (person left right))

## Lookup for binary trees

; lookup : number database $\rightarrow$ person or false ; to find the person with `ssn' in `bt' if possible (define (lookup ssn bt)
(cond
[(boolean? bt) false]
[else
(cond
[(ssn-matches? ssn (db-node-person bt)) (db-node-person bt)]
[else
(pick (lookup ssn
(db-node-left bt))
(lookup ssn
(db-node-right bt)))])]))

## Helper function for binary trees

; pick : person-or-false person-or-false $\rightarrow$ person-or-false
; to pick whichever, p1 or p2, that is a person,
; or false if none are
(define (pick p1 p2)
(cond

```
[(and (boolean? p1) (boolean? p2))
false]
    [(and (person? p1) (boolean? p2))
    p1]
    [(and (boolean? p1) (person? p2))
    p2]
    [(and (person? p1) (person? p2))
    p1]))
```

; ssn-matches?: ssn person $\rightarrow$ boolean
(define (ssn-matches? ssn person)
(= ssn (person-ssn person)))

```
(lookup =>
    5
    (make-bst
        (make-person 4 'tim)
        (make-bst
        (make-person 2 'jon)
        (make-bst
            (make-person 1 'anna)
            false
                false)
        (make-bst
            (make-person 3 'ellen)
            false
            false))
    (make-bst
        (make-person 6 'jane)
        (make-bst
            (make-person 5 'george)
            false
                false)
        (make-bst
            (make-person 7 'emily)
                false
                false))))
```

```
(lookup
    5
    (make-bst
        (make-person 4 'tim)
        (make-bst
        (make-person 2 'jon)
        (make-bst
            (make-person 1 'anna)
            false
                false)
        (make-bst
            (make-person 3 'ellen)
            false
            false))
    (make-bst
        (make-person 6 'jane)
        (make-bst
            (make-person 5 'george)
            false
                false)
        (make-bst
                (make-person 7 'emily)
                false
                false))) )
```

```
=> (pick
```

=> (pick
(lookup
(lookup
5
5
(make-bst
(make-bst
(make-person 2 'jon)
(make-person 2 'jon)
(make-bst
(make-bst
(make-person 1 'anna)
(make-person 1 'anna)
false
false
false)
false)
(make-bst
(make-bst
(make-person 3 'ellen)
(make-person 3 'ellen)
false
false
false)))
false)))
(lookup
(lookup
5
5
(make-bst
(make-bst
(make-person 6 'jane)
(make-person 6 'jane)
(make-bst
(make-bst
(make-person 5 'george)
(make-person 5 'george)
false
false
false)
false)
(make-bst
(make-bst
(make-person 7 'emily)
(make-person 7 'emily)
false
false
false))))

```
        false))))
```

```
<= (pick
(lookup
5
    (make-bst
                (make-person 2 'jon)
                (make-bst
                    (make-person 1 'anna)
                    false
                    false)
            (make-bst
                (make-person 3 'ellen)
                    false
                false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
                false
                false)
            (make-bst
                (make-person 7 'emily)
                false
                false))))
```

```
(pick
    (pick
        (lookup
            5
            (make-bst
                            (make-person 1 'anna)
                    false
                    false))
        (lookup
            5
            (make-bst
                    (make-person 3 'ellen)
                    false
                    false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                    (make-person 5 'george)
                false
                false)
            (make-bst
                (make-person 7 'emily)
                false
                false))))
```

```
<= (pick
```

<= (pick
(lookup
(lookup
5
5
(make-bst
(make-bst
(make-person 2 'jon)
(make-person 2 'jon)
(make-bst
(make-bst
(make-person 1 'anna)
(make-person 1 'anna)
false
false
false)
false)
(make-bst
(make-bst
(make-person 3 'ellen)
(make-person 3 'ellen)
false
false
false)) )
false)) )
(lookup
(lookup
5
5
(make-bst
(make-bst
(make-person 6 'jane)
(make-person 6 'jane)
(make-bst
(make-bst
(make-person 5 'george)
(make-person 5 'george)
false
false
false)
false)
(make-bst
(make-bst
(make-person 7 'emily)
(make-person 7 'emily)
false
false
false)) )

```
        false)) )
```

```
(pick
=>
    (pick
        (lookup
            5
            (make-bst
                (make-person 1 'anna)
                    false
                    false))
        (lookup
            5
            (make-bst
                    (make-person 3 'ellen)
                    false
                    false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
                false
                false)
            (make-bst
                (make-person 7 'emily)
                false
                false))))
```

```
(pick
    (pick
        (lookup
            5
            (make-bst
                            (make-person 1 'anna)
                    false
                    false))
        (lookup
            5
            (make-bst
                    (make-person 3 'ellen)
                    false
            false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
                false
                false)
            (make-bst
                (make-person 7 'emily)
                false
                false))))
                    => (pick
    (pick
        (pick
            (lookup 5 false)
            (lookup 5 false))
        (lookup
            5
            (make-bst
                (make-person 3 'ellen)
                    false
                    false)))
    (lookup
    5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
                    false
            false)
            (make-bst
                    (make-person 7 'emily)
                    false
            false))))
```

$$
<=\text { (pick }
$$

(pick
(lookup 5 false)
(lookup 5 false))
(lookup
5
(make-bst
(make-person 3 'ellen)
false
false)) )
(lookup
5
(make-bst
(make-person 6 'jane)
(make-bst
(make-person 5 'george)
false
false)
(make-bst
(make-person 7 'emily)
false
false)) )


```
(pick
=>
    (pick
        (pick false (lookup 5 false))
        (lookup
            5
            (make-bst
                    (make-person 3 'ellen)
                    false
                    false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                    (make-person 5 'george)
                    false
                    false)
            (make-bst
                    (make-person 7 'emily)
                    false
                    false))))
```

```
(pick => (pick
    (pick
        (pick false (lookup 5 false))
        (lookup
        5
            (make-bst
                    (make-person 3 'ellen)
                    false
                    false)))
    (lookup
        5
        (make-bst
                (make-person 6 'jane)
                (make-bst
                    (make-person 5 'george)
                    false
                    false)
                (make-bst
                    (make-person 7 'emily)
                    false
                false))))
```

```
    (pick
```

    (pick
        (pick false false)
        (pick false false)
        (lookup
        (lookup
        5
        5
            (make-bst
            (make-bst
                    (make-person 3 'ellen)
                    (make-person 3 'ellen)
                    false
                    false
                false)))
                false)))
    (lookup
    (lookup
        5
        5
        (make-bst
        (make-bst
            (make-person 6 'jane)
            (make-person 6 'jane)
            (make-bst
            (make-bst
                (make-person 5 'george)
                (make-person 5 'george)
                false
                false
                false)
                false)
            (make-bst
            (make-bst
                (make-person 7 'emily)
                (make-person 7 'emily)
                false
                false
                false))))
    ```
                false))))
```

```
<= (pick
    (pick
    (pick false false)
    (lookup
        5
        (make-bst
                    (make-person 3 'ellen)
                    false
                    false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
                    false
                false)
            (make-bst
                (make-person 7 'emily)
                    false
                false))))
```

| (pick (pick | $<=\begin{aligned} & \text { (pick } \\ & \text { (pick } \end{aligned}$ |
| :---: | :---: |
| false | (pick false false) |
| (lookup | (lookup |
| 5 | 5 |
| (make-bst | (make-bst |
| (make-person 3 'ellen) | (make-person 3 'ellen) |
| false | false |
| false)) | false)) |
| (lookup | (lookup |
| 5 | 5 |
| (make-bst | (make-bst |
| (make-person 6 'jane) | (make-person 6 'jane) |
| (make-bst <br> (make-person 5 'george) | (make-bst <br> (make-person 5 'george) |
| false | false |
| false) | false) |
| (make-bst | (make-bst |
| (make-person 7 'emily) | (make-person 7 'emily) |
| false | false |
| false)) ) | false) ) ) |

```
(pick
=>
    (pick
        false
        (lookup
            5
            (make-bst
                    (make-person 3 'ellen)
                    false
                    false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                    (make-person 5 'george)
                    false
                    false)
            (make-bst
                (make-person 7 'emily)
                    false
                    false))))
```

| $\begin{aligned} & \text { (pick } \\ & \text { (pick } \end{aligned}$ | => (pick ${ }_{\text {(pick }}$ |
| :---: | :---: |
| false | false |
| (lookup | (pick |
| 5 | (lookup 5 false) |
| (make-bst | (lookup 5 false))) |
| (make-person 3 'ellen) | (lookup |
| false | 5 |
| false)) | (make-bst |
| (lookup | (make-person 6 'jane) |
| 5 | (make-bst |
| (make-bst | (make-person 5 'george) |
| (make-person 6 'jane) | false |
| (make-bst | false) |
| (make-person 5 'george) | (make-bst |
| false | (make-person 7 'emily) |
| false) | false |
| (make-bst | false)) ) |
| (make-person 7 'emily) |  |
| false |  |
| false))) |  |

```
<= (pick
    (pick
    false
        (pick
            (lookup 5 false)
            (lookup 5 false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
                    false
                    false)
                (make-bst
                    (make-person 7 'emily)
                    false
                    false))))
```



```
(pick
    =>
    (pick
        false
        (pick false (lookup 5 false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
            false
            false)
            (make-bst
                (make-person 7 'emily)
                false
                false))))
```

```
(pick => (pick
    (pick (pick false (pick false false))
        false
        (pick false (lookup 5 false)))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                    (make-person 5 'george)
                    false
                    false)
            (make-bst
                (make-person 7 'emily)
                false
                false))))
```

```
<= (pick
    (pick false (pick false false))
    (lookup
        5
        (make-bst
            (make-person 6 'jane)
            (make-bst
                (make-person 5 'george)
                false
                false)
            (make-bst
                    (make-person 7 'emily)
                    false
                    false))))
```

```
(pick
    (pick false false)
    (lookup
        5
        (make-bst
        (make-person 6 'jane)
        (make-bst
            (make-person 5 'george)
            false
            false)
            (make-bst
                (make-person 7 'emily)
                false
                false))))
<= (pick
    (pick false (pick false false))
    (lookup
        5
    (make-bst
    (make-person 6 'jane)
    (make-bst
        (make-person 5 'george)
        false
        false)
    (make-bst
        (make-person 7 'emily)
        false
        false))))
```

```
(pick
=>
    (pick false false)
    (lookup
        5
(make-bst
(make-person 6 'jane)
(make-bst
            (make-person 5 'george)
            false
            false)
            (make-bst
                    (make-person 7 'emily)
            false
            false))))
```

(pick
(pick false false)
(lookup
5
(make-bst
(make-person 6 'jane)
(make-bst
(make-person 5 'george)
false
false)
(make-bst
(make-person 7 'emily) false
false))) )

```
=> (pick
    false
    (lookup
        5
            (make-bst
                            (make-person 6 'jane)
(make-bst
            (make-person 5 'george)
            false
            false)
            (make-bst
            (make-person 7 'emily)
            false
            false))))
```

$$
<=\begin{gathered}
\text { (pick } \\
\\
\\
\text { false } \\
\text { (lookup }
\end{gathered}
$$

            5
            (make-bst
                    (make-person 6 'jane)
                    (make-bst
                    (make-person 5 'george)
                    false
                false)
            (make-bst
                    (make-person 7 'emily)
                    false
                    false)) )
    | (pick | $<=$ (pick |
| :---: | :---: |
| false | false |
| (pick | (lookup |
| (lookup | 5 |
| 5 | (make-bst |
| (make-bst | (make-person 6 'jane) |
| (make-person 5 'george) | (make-bst |
| false | (make-person 5 'george) |
| false)) | false |
| (lookup | false) |
| 5 | (make-bst |
| (make-bst | (make-person 7 'emily) |
| (make-person 7 'emily) | false |
| false false)) ) | false)) ) |

```
(pick
=>
false
    (pick
        (lookup
            5
            (make-bst
                    (make-person 5 'george)
                    false
                    false))
    (lookup
                5
            (make-bst
                    (make-person 7 'emily)
                    false
                    false))))
```

```
(pick
=> (pick
false
    (pick
        (lookup
            5
            (make-bst
                    (make-person 5 'george)
                    false
                    false))
        (lookup
            5
            (make-bst
                    (make-person 7 'emily)
                    false
                    false))))
    false
    (pick
        (make-person 5 'george)
                (lookup
            5
                    (make-bst
                    (make-person 7 'emily)
                    false
                    false))) )
```

```
<= (pick
    false
    (pick
        (make-person 5 'george)
        (lookup
            5
            (make-bst
                    (make-person 7 'emily)
                    false
                    false))))
```

```
(pick
false
    (pick
        (make-person 5 'george)
        (pick
            (lookup 5 false)
            (lookup 5 false))))
```

```
<= (pick
```

<= (pick
false
false
(pick
(pick
(make-person 5 'george)
(make-person 5 'george)
(lookup
(lookup
5
5
(make-bst
(make-bst
(make-person 7 'emily)
(make-person 7 'emily)
false
false
false))) )

```
                    false))) )
```

```
(pick
=>
false
    (pick
        (make-person 5 'george)
        (pick
            (lookup 5 false)
            (lookup 5 false))))
```

```
(pick
false
    (pick
        (make-person 5 'george)
        (pick
            (lookup 5 false)
            (lookup 5 false))))
```

$$
\begin{aligned}
<= & \text { (pick } \\
& \text { false } \\
& \text { (pick } \\
& \text { (make-person } 5 \text { 'george) } \\
& (\text { pick false (lookup } 5 \text { false)))) }
\end{aligned}
$$

```
(pick
false
(pick
        (make-person 5 'george)
        (pick false false)))
<= (pick
    false
    (pick
    (make-person 5 'george)
    (pick false (lookup 5 false))))
```

```
(pick
=>
false
(pick
(make-person 5 'george)
(pick false false)))
```

| (pick | $=>$ (pick |
| :---: | :---: |
| false | false |
| (pick | (pick |
| (make-person 5 'george) | (make-person 5 'george) |
| (pick false false)) | false)) |

$$
\begin{array}{rll}
<= & \text { (pick } \\
& \text { false } \\
& \text { (pick } \\
& \text { (make-person } 5 \text { 'george) } \\
& \text { false)) }
\end{array}
$$

| (pick <br> false <br> (make-person 5 | $<=$(pick <br> false <br> (pick <br> (make-person 5 <br> false)) |
| :--- | :--- |

```
(pick
=>
false
    (make-person 5 'george))
```

(pick
false
(make-person 5 'george))
(that took forever!)

## Binary search trees

; a database is
; - false
; - (make-db-node person database[left] database[right])
; where all people in left have smaller ssns than person, and
; all people in right have larger ssns than person.
(define-struct $d b$-node (person left right))

## Lookup for binary search trees

; lookup : number database $\rightarrow$ person or false
; to find the person with `ssn' in `bt' if possible (define (lookup ssn bt)
(cond
[(boolean? bt) false]
[else
(cond

```
[(ssn-matches? ssn (db-node-person bt))
        (db-node-person bt)]
    [(ssn-less? ssn (db-node-person bt))
        (lookup ssn (db-node-left bt))]
    [(ssn-more? ssn (db-node-person bt))
        (lookup ssn (db-node-right bt))])]))
```


## Helper functions for binary search trees

```
; ssn-less? : ssn person -> boolean
(define (ssn-less? ssn person)
    (< ssn (person-ssn person)))
; ssn-more? : ssn person -> boolean
(define (ssn-more? ssn person)
    (> ssn (person-ssn person)))
```

```
(lookup =>
    5
    (make-bst
        (make-person 4 'tim)
        (make-bst
        (make-person 2 'jon)
        (make-bst
            (make-person 1 'anna)
            false
                false)
        (make-bst
            (make-person 3 'ellen)
            false
            false))
    (make-bst
        (make-person 6 'jane)
        (make-bst
            (make-person 5 'george)
            false
                false)
        (make-bst
            (make-person 7 'emily)
                false
                false))))
```

```
(lookup
5
(make-bst
        (make-person 4 'tim)
        (make-bst
        (make-person 2 'jon)
        (make-bst
            (make-person 1 'anna)
            false
                false)
        (make-bst
                (make-person 3 'ellen)
            false
                false))
    (make-bst
        (make-person 6 'jane)
        (make-bst
                (make-person 5 'george)
                false
                false)
        (make-bst
                (make-person 7 'emily)
                false
                false))) )
                => (lookup
```

```
<= (lookup
5
(make-bst
    (make-person 6 'jane)
    (make-bst
            (make-person 5 'george)
            false
            false)
        (make-bst
            (make-person 7 'emily)
            false
            false)))
```

| ```(lookup 5 (make-bst (make-person 5 'george) false false))``` | ```<= (lookup 5 (make-bst (make-person 6 'jane) (make-bst (make-person 5 'george) false false) (make-bst (make-person 7 'emily) false false)))``` |
| :---: | :---: |

(make-bst
(make-person 5 'george)
false
false))
(lookup
5
(make-bst
(make-person 5 'george)
false
false))

## (much quicker that time, no?)

